Uniformly optimal prediction of infinite words

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Abstract

The present paper links the concepts of Kolmogorov complexity (in Complexity theory) and Hausdorff dimension (in Fractal geometry) for a class of recursive (computable) ω-languages.

It is shown that the complexity of an infinite string contained in a Σ_2-definable set of strings is upper bounded by the Hausdorff dimension of this set and that this upper bound is tight. Moreover, we show that there are computable gambling strategies guaranteeing a uniform prediction quality arbitrarily close to the optimal one estimated by Hausdorff dimension and Kolmogorov complexity provided the gambler's adversary plays according to a sequence chosen from a Σ_2-definable set of strings.

We provide also examples which give evidence that our results do not extend further in the Arithmetical hierarchy.