# A Modal $\mu$–Calculus for Durational Transition Systems

Helmut Seidl
FB IV – Informatik
Universität Trier
D–54286 Trier
Germany
email: seidl@ti.uni-trier.de

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**Abstract**

Durational transition systems are finite transition systems where every transition is additionally equipped with a duration. We consider the problem of interpreting $\mu$–formulas over durational transition systems. In case the formula contains only operations minimum, maximum, addition, and sequencing, we show that the interpretation ist not only computable but (up to a linear factor) as efficiently computable as the interpretation of ordinary $\mu$–formulas over finite transition systems.

**Keywords:** $\mu$–Calculus, durational transition systems, model checking.

## 1 Introduction

The $\mu$–calculus, a propositional modal logic with fixpoint operators, was introduced by Dexter Kozen as a tool for the description and analysis of the behavior of possibly infinite computations [Ko83]. The expressive power of modal $\mu$–calculus is intimately connected to finite–state automata on infinite trees [VW86, EJ91, EJS93, BVW94]. Classically, a $\mu$–formula denotes a predicate on states. Typical properties to be expressed and analyzed are *safety* and *lifeness* assertions. The formula $\nu x.\langle a\rangle x$, for example, denotes the set of all states allowing for an infinite sequence of $a$–actions. Especially, the modal operator $\langle a\rangle$ (for action $a$) constructs a property of the actual state from a property of a next state. Thus, it relates presence to a (possibly infinite) future. For detailed explanation of various modal (and temporal) logics consult, e.g., Colin Stirling in [St92].

The problem with which we are concerned here, however, is not only to determine properties of states but also quantitative information. We are interested in questions like: how long does it take a process to reach a state with a certain property. Technically, this means that closed formulas are no longer interpreted as predicates or, equivalently, as mappings $Q \to \{0, 1\}$ for some base set $Q$. Instead, we employ mappings $t : Q \to D$ for some time domain $D$. In this paper we consider $D$ as a subinterval of

$$-\infty < 0 < 1 < 2 < \ldots < n < \ldots < \infty$$

i.e., the non–negative integers extended by $-\infty$ (to express unaccessibility) and $\infty$. The "logical" connectives "$\vee$ and "$\wedge$" are now replaced with maximum and minimum, respectively. Additionally, we consider addition "+" (appropriately extended) and the sequencing operator ";" which returns the second argument only provided the first one is different from $-\infty$. Otherwise, the result is $-\infty$. While addition is useful for modelling the time behavior of two processes being executed one after the other, the ";" may serve for a primitive form of conditional: the second argument is only counted provided the event corresponding to the first will ever occur.

A corresponding theory of processes with durational actions was developed by Gorrieri, Roccetti and Stancampiano [GRS95]. For the analysis of complexity-like properties of programs, e.g., the number of accesses to some variable, least solutions of (non–hierarchical) systems of equations over non–negative integers (extended by $\infty$) have been considered in [Se94]. This corresponds to the interpretation of $\mu$–formulas without occurrences of the greatest fixpoint operator.

In this paper we are going to investigate whether or not the interpretation of arbitrary $\mu$–formulas can be effectively computed – even when the chosen time interval is infinite. It turns out that this is in deed the case. Moreover, we find that it is not only computable but, even more, as efficiently (at least up to a linear factor) computable as the interpretation over a two point interval.

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