On language families of interacting automata

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The language recognition capabilities of some classes of interacting automata are studied. Classical one-dimensional *cellular spaces* and various modifications and generalizations have been investigated for a long time (e.g. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13]). We restrict our considerations on some of such classes Smith [12] collectively called *polyautomata*, their iterative variants and a real generalization obtained by stack augmentation. Since in the theory of polyautomata there are open problems, as are the question whether linear-time *cellular automata* are more powerful than real-time cellular automata or not, we designate real-time acceptors and acceptors without any time restrictions.

At first we summarize the known relationships between some of the language families. Subsequently the languages definable by the iterative variant of *pushdown cellular automata* are investigated.

Definition.
An *iterative pushdown array* (IPDA) is a system $(S, \Sigma, \Gamma, \sigma_0, \sigma, q_0, g_0, \texttt{\#})$, where

a) S is the finite, nonempty set of *states*,
b) Σ is the finite, nonempty set of *input symbols*,
c) Γ is the finite, nonempty set of *stack symbols*,
d) $g_0 \in \Gamma$ is the *bottom of stack symbol*,
e) $\texttt{\#} \in \Sigma$ is the *end of input symbol*,
f) $\sigma_0 : S^3 \times \Gamma \times \Sigma \longrightarrow S \times \Gamma^*$ and
$\sigma : S^3 \times \Gamma \longrightarrow S \times \Gamma^*$
are the *local transformations* of the cell at the origin and of all other cells respectively, satisfying

$\forall s_1, s_2, s_3, s_4 \in S, g \in \Gamma, a \in \Sigma : \ \sigma(s_1, s_2, s_3, g) = (s_4, \gamma) \Longrightarrow$
(resp. $\sigma_0(s_1, s_2, s_3, a, g) = (s_4, \gamma) \Longrightarrow$)
$(\gamma \in (\Gamma \setminus \{g_0\})^* \ \wedge \ g \neq g_0) \ \vee \ (\gamma = \gamma' g_0 \ \wedge \ \gamma' \in (\Gamma \setminus \{g_0\})^* \ \wedge \ g = g_0)$

g) $q_0 \in S$ is the *quiescent state*, such that for all $g \in \Gamma \quad \sigma(q_0, q_0, q_0, g) = (q_0, g)$ holds.

The local transition functions induce a length preserving mapping (*global transformation*) $\mathcal{T} : (S \times \Gamma^+)^+ \times \Sigma^\omega \longrightarrow (S \times \Gamma^+)^+ \times \Sigma^\omega$ according to the following.

Let $\bar{\sigma} : S^3 \times \Gamma^+ \longrightarrow S \times \Gamma^+$ be defined as

$$\bar{\sigma}(s_1, s_2, s_3, g_m \cdots g_0) := \Big(\pi_1(\sigma(s_1, s_2, s_3, g_m)), \pi_2(\sigma(s_1, s_2, s_3, g_m)) g_{m-1} \cdots g_0\Big),$$

and $\bar{\sigma}_0 : S^3 \times \Gamma^+ \times \Sigma \longrightarrow S \times \Gamma^+$ be defined as

$$\bar{\sigma}_0(s_1, s_2, s_3, g_m \cdots g_0, a) := \Big(\pi_1(\sigma_0(s_1, s_2, s_3, g_m, a)), \pi_2(\sigma_0(s_1, s_2, s_3, g_m, a)) g_{m-1} \cdots g_0\Big),$$

then

$\forall m \in \mathbb{N}, i \in \{-m, \ldots, -1, 0, 1, \ldots, m\} : \ \forall s_i \in S, \ \gamma_i \in \Gamma^+, \ a \in \Sigma :$

$$\mathcal{T}((s_{-m}, \gamma_{-m}) \ldots (s_0, \gamma_0) \ldots (s_m, \gamma_m), a a_1 a_2 \cdots) :=$$
$$\begin{cases} (\bar{\sigma}(q_0, q_0, s_0, g_0)\bar{\sigma}_0(q_0, s_0, q_0, \gamma_0, a)\bar{\sigma}(s_0, q_0, q_0, g_0), a_1 a_2 \cdots) & \text{if } m = 0 \\ (\bar{\sigma}(q_0, q_0, s_{-m}, g_0) \cdots \bar{\sigma}_0(s_{-1}, s_0, s_1, \gamma_0, a) \cdots \bar{\sigma}(s_m, q_0, q_0, g_0), a_1 a_2 \cdots) & \\ & \text{if } m > 0 \end{cases}$$

Definition.
Let $M = (S, \Sigma, \Gamma, \sigma_0, \sigma, q_0, g_0, \#)$ be an IPDA, $L \subseteq (\Sigma \setminus \{\#\})^+$ a formal language and $t : \mathbb{N} \longrightarrow \mathbb{N}$ a function.
L is *accepted* by M in time t (with respect to F) if and only if $F \subseteq S$ and $L = \{w | \, \pi_1(\pi_0(\mathcal{T}^{t(|w|)}((q_0, g_0), w\#^\omega))) \in F \wedge \forall t' < t(n) : \mathcal{T}^{t'}((q_0, g_0), w\#^\omega))) \notin F\}$ holds.

The families of languages acceptable by IPDAs in time t are denoted by $\mathscr{L}_t(\text{IPDA})$. Of special interest are the real-time languages which can be accepted within time $rt(n) := n$.

Trivially, $\mathscr{L}_t(\text{IA}) \subseteq \mathscr{L}_t(\text{IPDA})$ holds true.
We can prove the following results:

Theorem. There exists a context-free language not acceptable by IPDAs in real-time.

Theorem. There exists a context-sensitive not context-free language which belongs to $\mathscr{L}_{rt}(\mathrm{IPDA})$.

We conclude that the families $\mathscr{L}_2$ and $\mathscr{L}_{rt}(\mathrm{IPDA})$ are incomparable.

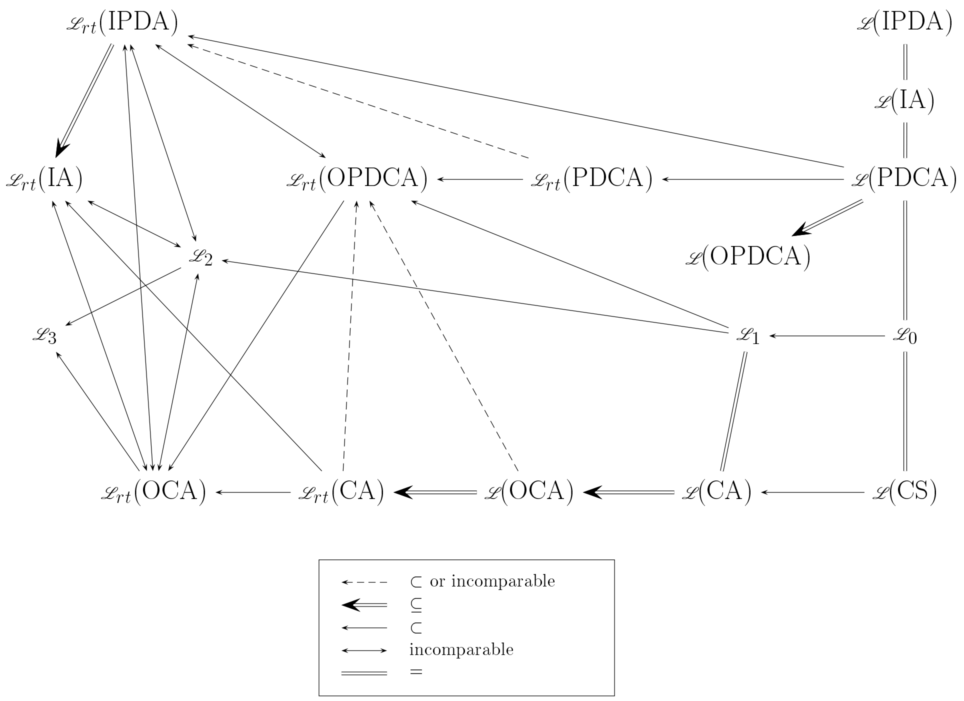
Theorem. $\exists L \notin \mathscr{L}_{rt}(\mathrm{OPDCA}) : \ L \in \mathscr{L}_{rt}(\mathrm{IPDA})$.

Theorem. $\exists L \in \mathscr{L}_{rt}(\mathrm{OPDCA}) : \ L \notin \mathscr{L}_{rt}(\mathrm{IPDA})$.

We conclude that the families $\mathscr{L}_{rt}(\mathrm{OPDCA})$ and $\mathscr{L}_{rt}(\mathrm{IPDA})$ are incomparable.

Theorem. The time t online TM languages are a subset of $\mathscr{L}_t(\mathrm{IPDA})$.

In the following figure some relationships are depicted.



References

[1] Beyer, W. T. *Recognition of topological invariants by iterative arrays.* Technical Report MAC TR-66, Massachusetts Institute of Technology, 1969.

[2] Bucher, W. and Culik II, K. *On real time and linear time cellular automata.* RAIRO Informatique Théorique et Applications. 18 (1984), 307–325.

[3] Choffrut, C. and Culik II, K. *On real-time cellular automata and trellis automata.* Acta Informatica 21 (1984), 393–407.

[4] Cole, S. N. *Real-time computation by n-dimensional iterative arrays of finite-state machines.* IEEE Conference Record of 7^{th} Annual Symposium on Switching and Automata Theory, 1966, pp. 53–77.

[5] Dyer, C. R. *One-way bounded cellular automata.* Information and Control 44 (1980), 261–281.

[6] Kosaraju, S. R. *On some open problems in the theory of cellular automata.* IEEE Transactions on Computers C-23 (1974), 561–565.

[7] Kutrib, M. *Real-time language recognition by pushdown cellular automata.* Proc. Workshop on Parallel Processing, Lessach, 1994, pp. 126–140.

[8] Kutrib, M. and Richstein, J. *Real-time one-way pushdown cellular automata languages.* Bericht 9403, Arbeitsgruppe Informatik, Universität Gießen, Gießen, 1994.

[9] Kutrib, M. and Worsch, Th. *Investigation of different input modes for cellular automata.* Parcella '94, Akademie Verlag, Berlin, 1994, pp. 141–150.

[10] Seidel, S. R. *Language recognition and the synchronization of cellular automata.* Technical Report 79-02, Department of Computer Science, University of Iowa, Iowa City, 1979.

[11] Smith III, A. R. *Real-time language recognition by one-dimensional cellular automata.* Journal of Computer and System Sciences 6 (1972), 233–253.

[12] Smith III, A. R. *Introduction to and survey of polyautomata theory.* In Lindenmayer, A. and Rozenberg, G. (eds.), *Automata, Languages, Development.* North-Holland, Amsterdam, 1976, pp. 405–422.

[13] Vollmar, R. *Algorithmen in Zellularautomaten.* Teubner, Stuttgart, 1979.