$5' \longrightarrow 3'$ Watson-Crick Automata with Several Runs

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Joint work with Benedek Nagy (Debrecen)
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Motivation for Watson-Crick Automata

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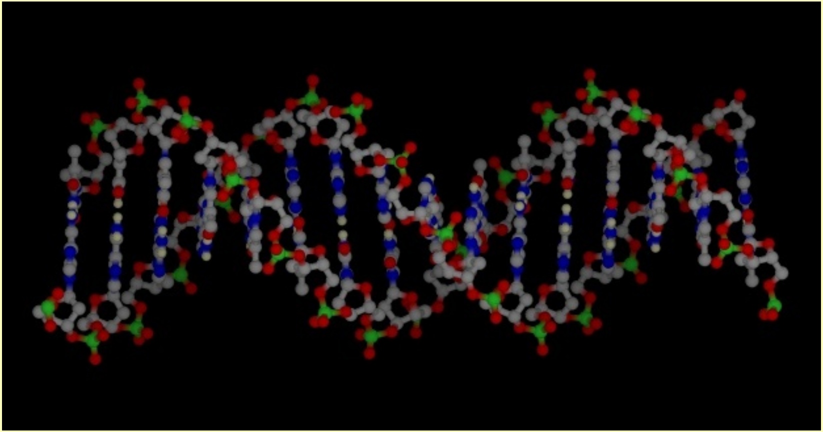
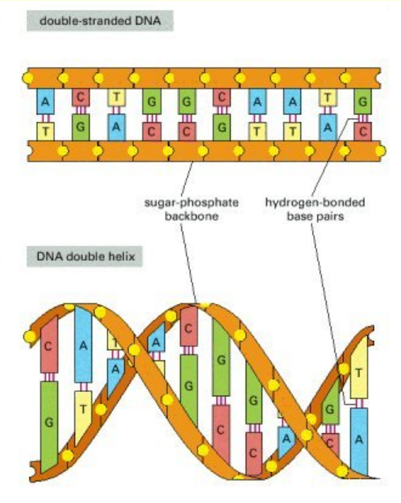
A DNA strand is not just a simple string, but normally is a double strand with a three-dimensional helix structure.



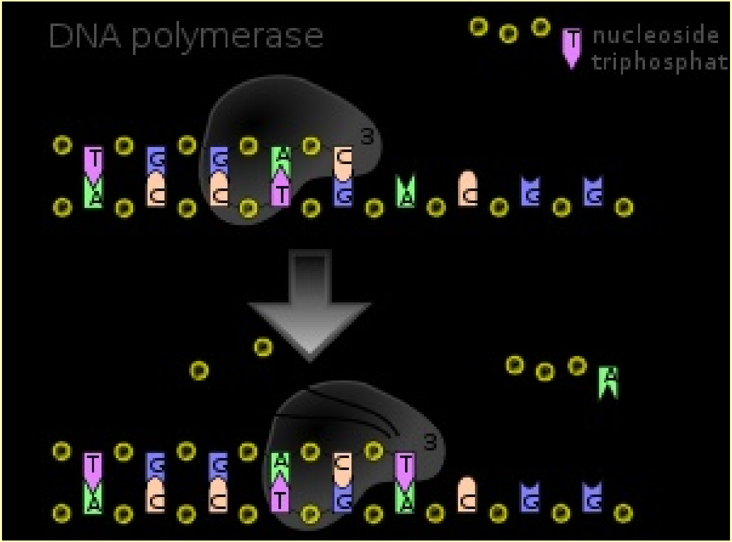
Abstracting the Structure

We view a DNA strand as a linear sequence of pairs of complementary symbols..



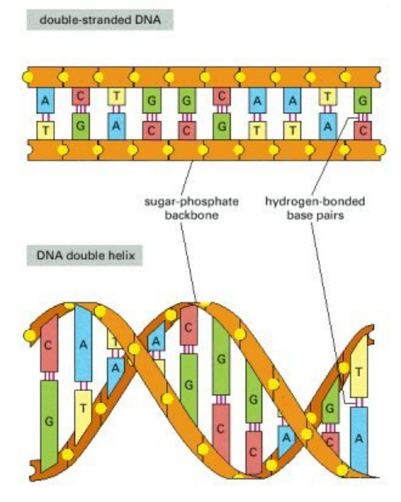
The Head

What could an automaton's reading head look like?



Two Heads

To read both parts of the double strand, two heads are necessary.



The Role of Complementarity

Intuitively, a strand and its complement are equivalent from an information theoretic point of view.

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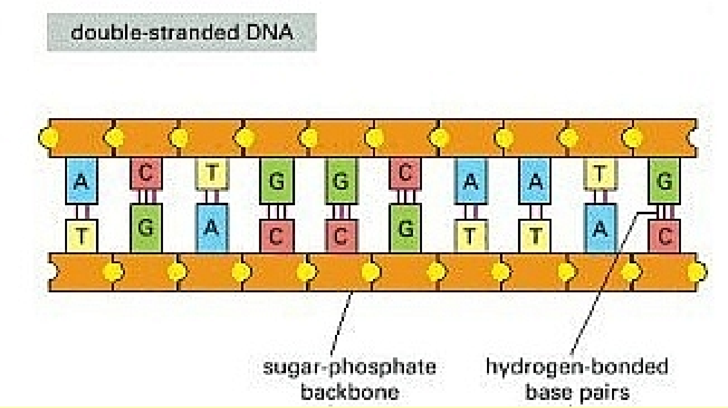
Therefore we will let the automata's two heads work on the same string for simplicity of notation.

Moving the Heads

The two strands have a direction...



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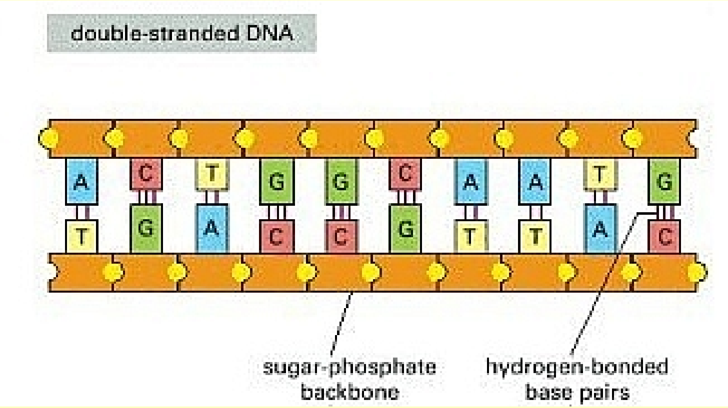

...and their directions are opposite.

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So in contrast to conventional Watson-Crick automata we can expect ease in recognizing palindromic structures and problems in recognizing copy-type structures.

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An input word is accepted in k runs, iff the automaton halts in an accepting state after k runs.

Variants

Definition

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The corresponding classes of languages are denoted by $U\overset{\rightleftharpoons}{k}\mathbf{WK}$ where U is one of the symbols associated to the variants in the enumeration above. Also combinations of these variants are possible.

Lemma

Let A be a deterministic $5' \rightarrow 3'$ WK-automaton accepting the language $L := \{ba^nba^nb : n > 0\}$ in one run. For long enough n the computation for an input word ba^nba^nb goes through a configuration where one head is in the first factor a^n while the other is in the second.

Behaviour of $5' \longrightarrow 3'$ Watson-Crick Automata

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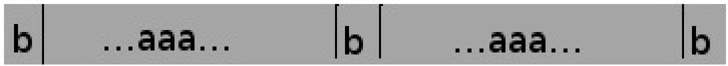

An Infinite Hierarchy of $5' \longrightarrow 3'$ Watson-Crick Automata

Theorem

For every $m > 0$ the class of languages accepted by $5' \rightarrow 3'$ deterministic WK-automaton in m runs is properly contained in the class of languages accepted in $m + 1$ runs, i.e., $\mathbf{D}_{m}^{\rightleftharpoons}\mathbf{WK} \subsetneq \mathbf{D}_{m+1}^{\rightleftharpoons}\mathbf{WK}$.

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The witness languages are:

$$L_m := \{ww : w \in L'_m\}$$

where

$$L'_m := \{a^{n_1} b^{n_2} a^{n_3} b^{n_4} \ldots a^{n_{2m-1}} b^{n_{2m}} : n_i > 0 \text{ for } 1 \leq i \leq 2m\}.$$

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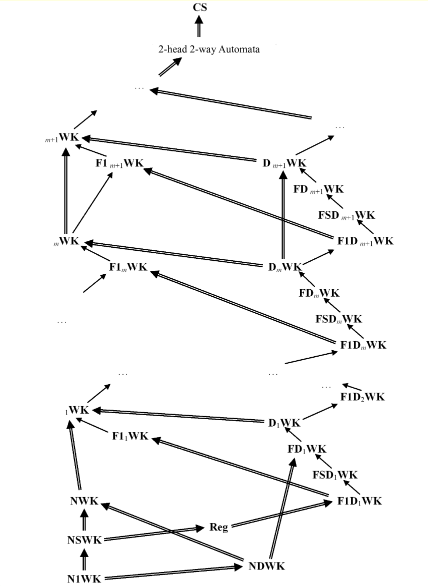

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However, since they do not accept all context-free languages, but language classes somewhat orthogonal to the Chomsky Hierarchy, things are not obvious.

The Emptiness Problem

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For a Turing Machine M we define the language L_M that contains all words $w_1\#w_2\ldots\#w_{i-1}\#\#w_i^R\#w_{i-1}^R\ldots\#w_3^R\#w_2^R$ where $w_1, w_2, \ldots, w_i$ is a sequence of configurations of a computation of M, w_1 is an initial configuration, and w_i is a final and accepting configuration.

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- read w_1 and w_2^R simultaneously
- check whether w_i and w_i^R are really the same
- accepts iff the string represents an accepting TM computation

Corollaries

Corollary

For the class $\mathbf{D}_1^{\rightleftharpoons}\mathbf{WK}$ the finiteness problem is undecidable.

A deterministic TM accepts words in only one possible computation. Therefore L_M is finite iff M's language is finite.

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Corollary

Every recursively enumerable language is a morphic image of a language from $\mathbf{D}_1^{\rightleftharpoons}\mathbf{WK}$.

The word accepted by a computation can be extracted from L_m by a simple morphism.

Open Problems

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We have establishes the undecidability of the emptiness problem for deterministic $5' \rightarrow 3'$ WK-automata with one run. Close to the bottom of our hierarchy, the regular languages appear. By definition we have the inclusions $\mathbf{F1D}_{m}^{\rightleftharpoons}\mathbf{WK} \subseteq \mathbf{FSD}_{m}^{\rightleftharpoons}\mathbf{WK} \subseteq \mathbf{FD}_{m}^{\rightleftharpoons}\mathbf{WK} \subseteq \mathbf{D}_{m}^{\rightleftharpoons}\mathbf{WK}$ on the path between them, but as mentioned above, it is unclear, which of them are proper.

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Open Problem

Secondly, we believe that $\mathbf{D}_1^{\rightleftharpoons}\mathbf{WK}$ *is incomparable to the class of linear languages.*

There are examples for $\mathbf{D}_1^{\rightleftharpoons}\mathbf{WK} \subsetneq LIN$*, but no example for* $\mathbf{D}_1^{\rightleftharpoons}\mathbf{WK} \supsetneq LIN$*.*