On parallel Turing machines with multi-head control units

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Parallel Turing machines

HEMMERLING (1979) was probably the first to consider a model where several finite automata are working on one common tape cooperatively. Later WIEDERMANN (1984) generalized the model to so-called *parallel Turing machines* (PTM) which consist of a fixed finite number of tapes with a (possibly varying) number of Turing machine like control units (CU) working on it cooperatively. The CU are identical, i.e. they all have the same number of heads on each tape and work according to the same rule, which includes the ability to delete itself or generate additional CUs.

While one-tape one-head PTM are linear-time and linear-space equivalent to one-dimensional cellular automata and hence in the first machine class $\mathcal{C}_1$ (see VAN EMDE BOAS, 1990), it has been argued by WIEDERMANN (1992) that multi-head PTM are not. It turns out that this essentially depends on the definition of space complexity s for PTM, namely $s = r_t$ where $r_t(n)$ is the number of tape cells used by the PTM for inputs of length n.

We suggest another definition, $s = r_t + \log \binom{r_t^h}{r_h}$ where h is the number heads of the CU and r_h is the maximum number of CU simultaneously working on the tapes (WORSCH, 1996). This definition is motivated by the analogous problem for the *storage modification machines* of SCHÖNHAGE (1980) as discussed e.g. by VAN EMDE BOAS (1989).

PTM versus other models

Using the new definition of space complexity for PTM one can describe simulations by CA and vice versa which imply that PTM *are* in $\mathcal{C}_1$.

WIEDERMANN (1992) observed that it is possible to distinguish different types of models within $\mathcal{C}_1$ and introduced the notion of *weak parallel machines*. These are characterized by their ability to process whole *sequences of inputs* in

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a pipelined fashion. Cellular automata are an example of a weak parallel model while sequential Turing machines probably aren't.

We extend this idea a little bit further by having a look specifically at *"really long" sequences of inputs* (for some precise meaning of "really long"). Surprisingly it turns out that in some sense multi-head PTM probably are even more powerful than cellular automata and not a weak parallel model.

Informally one might say, that multi-head PTM are in the first machine class for short input sequences and in the second machine class for long input sequences.

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