CA simulations and observations in biological and geological processes. Is there evidence for class 4 in nature?

Ingo Kusch[1], Mario Markus[1], Pedro Almeida[2] and António Ribeiro[2]

[1]Max-Planck-Institut für molekulare Physiologie, Postfach 102664, 44026 Dortmund, Germany

[2]Departamento de Geologia, Faculdade de Ciências, Universidade de Lisboa;
58 rua da Escola Politécnica, P-1200 Lisboa, Portugal

We developed CA with binary output for simulations of: i) the pigmentation of mollusc shells; ii) stick-slip processes in earthquake dynamics; and iii) two- and three-dimensional Turing patterns.

The CA for shell pigmentation and for Turing patterns were constructed via a one-to-one assignment of CA rules to interaction arrows in a system based on Turing's [1] morphogenetic model.

Excellent agreement of CA simulations is obtained for a large number of natural mollusc shells. The patterns can be classified using Wolfram's nomenclature (classes 1 through 4). In particular, class 4 CA agree with mixed patterns (aperiodic alternation of periodicity and chaos) in real shells. This is remarkable for two reasons: i) it may by an indication for the existence of class 4 in nature; and ii) we obtain class 4 with the same rules leading to other classes, as opposed to the approach with PDEs [2] in which special systems of up to 6 PDEs had to be introduced just to simulate the cases of mixed patterns.

A binary version of a Burridge-Knopoff-type model [3,4] for stick-slip processes in geophysical plate-displacements also yields CA classes 1 through 4. There is agreement of CA simulations with the measured Gutenberg-Richter law, as well as with the observation of a characteristic seism. We find a high sensitivity of the results with respect to the energy dissipation p, transitions between CA classes occuring back and forth within very small p-intervalls. This implies an additional uncertainty for predictions in this system.

Two-dimensional Turing-patterns have since long been assumed to give positional information for genetic pattern-determination in skins and furs. We find transitions from stripes to spots for increasing inhibitory parameters, as well as transitions from squared to hexagonal to disorded patterns for increasing system size. Size-dependence was also obtained in a generalization to three dimensions; here, the CA yielded patterns that can be classified cristallographically, as in solid-state-physics: bcc, fcc, hexagonal, or disorded lattices of spots.

In our biological and geological CA simulations we have well defined control-parameters - e.g. the energy dissipation p - which allow bifurcation analysis. We thus do not have the problem encountered in the parametrization of Wolfram's automata (see [5]), in which the parameter λ has no physical meaning, nor is it associated with a unique set of rules.

[1] Turing, A. Phil.Trans.Roy.Soc. B237, 37-72 (1952)
[2] Meinhardt, H., The Algorithmic Beauty of Sea Shells, Springer, Berlin (1995)
[3] Burridge, R. and Knopoff, L., Bull.Sei.Soc.Am. 57, 341-371 (1967)
[4] Bak, P. and Tang, C., J.Geophys.Res. 94, 15.635-15.637 (1989)
[5] Li, W., Packard, N.H. and Langton, C.G., Physica D45, 77-94 (1990)