Towards a new approach to universality about Turing machines

Maurice MARGENSTERN and Lioudmila PAVLOTSKAÏA
LITP/IBP,
LRIM (Metz University),
Metz University Institute of Technology

Moscow Institute
of Power Engeneering

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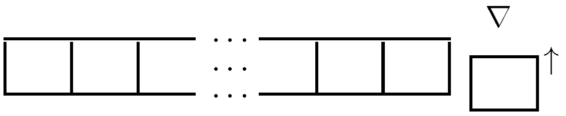
Our talk will introduce a new approach towards universality in the field of classical Turing macines, *i.e.* deterministic Turing machines with a single head and a single tape, infinite in both directions.

This approach is motivated by the relatively small size of universal Turing machines obtained in the eighties by Yu. Rogozhin, see [4]. The smallest known classical Turing machine there appeared as a machine with only 23 instructions.

How can this number be lowered?

1. A new notion

Our basic idea, see [2], consists in setting together a Turing machine with a finite automaton in the following way. Consider the initial datum of Turing machine M as a word w. After a certain time of computation, if the machine did not reach two indentical configurations inside the initial segment I delimited by w at initial time t_0, then its head makes an attempt to read the next symbol *outside* of I. If this attempt is made on the left side, we consider that the machine halts. If it is made on the right side, we consider that the asked letter is supplied by a given finite automaton, say A, as illustrated by the following figure.



Notice that A always provides the blank symbol, a classical machine is obtained. And so, this new notion generalizes classical Turing computations.

2. The results

Rogozhin's machines are constructed according to the tag-system technique introduced in the last fifties and early sixties by several people.

In [3], a universal Turing machine is constructed with 17 instructions thanks to its association with a finite automaton which produces a periodic infinite word. Below is given the program of this machine, say F. It should be noticed that machine F does not simulate a tag-system but a two-register machine, following the simulation pattern defined in [1] in order to construct universal classical Turing machines with only two left instructions.

Below also, is the program of machine G which, associated with a suitable finite automaton, is able to simulate any classical Turing machine. Machine G is thus universal. It has only eight instructions:

	0	1	2	3	4	5	6
1	$5\,R$	$4\,R\,2$	$0\,L$	$5\,R$	$1\,L$	$0\,L$	$R\,3$
2	$5\,R$	$4\,R$		$5\,R\,4$			
3	$5\,R$	R	$5\,R\,2$	$5\,R$			
3	R		$L\,1$	$1\,R$			

	0	1	2	3	4
1	$1\,L$	$0\,R$	$R\,2$	$4\,L$	$3\,R$
2		R		$4\,L\,1$	$3\,R$

By loosing a bit more the simulation performed by machine G, a machine with only *five* instructions, say H can be constructed which is universal when associated with a suitable finite automaton. Here is its table:

	0	1	2
1	$1\,L$	$0\,R$	$R\,2$
2		R	$1\,L\,1$

In [2], tables for automata associated to, respectively, machines G and H are provided with full proofs of the correctness of the computation starting from an appropriate encoding of the register initial contents.

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