Firing Squad Synchronization Problem in One and Two Dimensional Reversible Cellular Automaton

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Abstract

The Firing Squad Synchronization Problem (FSSP) is to construct a one-dimensional three-neighbor cellular automata of arbitrary finite length such that one of the end cell (general) makes all the other cells (soldiers) be in a particular state (firing state) at a certain time. The problem was first solved by Minsky and McCarthy. They constructed a solution synchronizing n cells in $3n$ steps using a divide-and-conquer method. At present 6-state minimal time solution has been given by Mazoyer[1]. Two-dimensional case was studied by Rosenstiehl, and Kobayashi showed some special classes of figures that have "linear" firing time [2].

We studied FSSP on reversible (i.e., backward deterministic) cellular automata (RCA). First we proved that, in the case of RCA, there is no solution under the usual condition for FSSP where the firing state is only one. So we defined a little weaker condition suitable for RCA which is as follows.

[A synchronizing condition for one-dimensional RCA]

1. Define a set $F(\subset Q)$ of finite number of firing states, and regard that the cells synchronize if the state of each cell is in F.

2. Not only the general and soldier cells but the other cells are allowed to change its internal states.

Under this condition, we showed that $3n$ time solution can be embedded in an RCA with 99 states that satisfies the above condition (Fig.2). Constructing solution, we used the framework of partitioned CA (PCA)[3], which can be regarded as a subclass of CA. It has been proved that local function of a PCA(Fig.1) is one-to-one iff their global function is one-to-one (i.e. reversible). So we can construct a RCA easily.

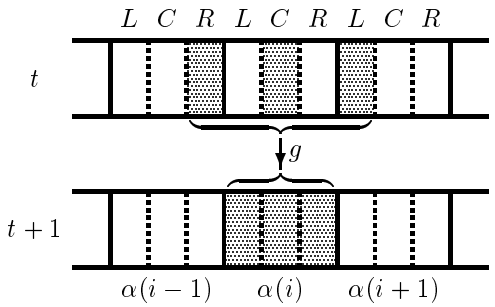

Figure 1: Domain and range of a local function of one-dimensional PCA.

The same approach can be usable for two-dimensional cases. We extend the synchronizing condition into two-dimensional RCA and embeded the solutions of Kobayashi and Rosenstiehl in two-dimensional reversible PCA.

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References

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[3] Morita, K., and Harao, M.: Computation universality of one-dimensional reversible (injective) cellular automata, *Trans. IEICE Japan*, **E72**, (1989) 758–762.

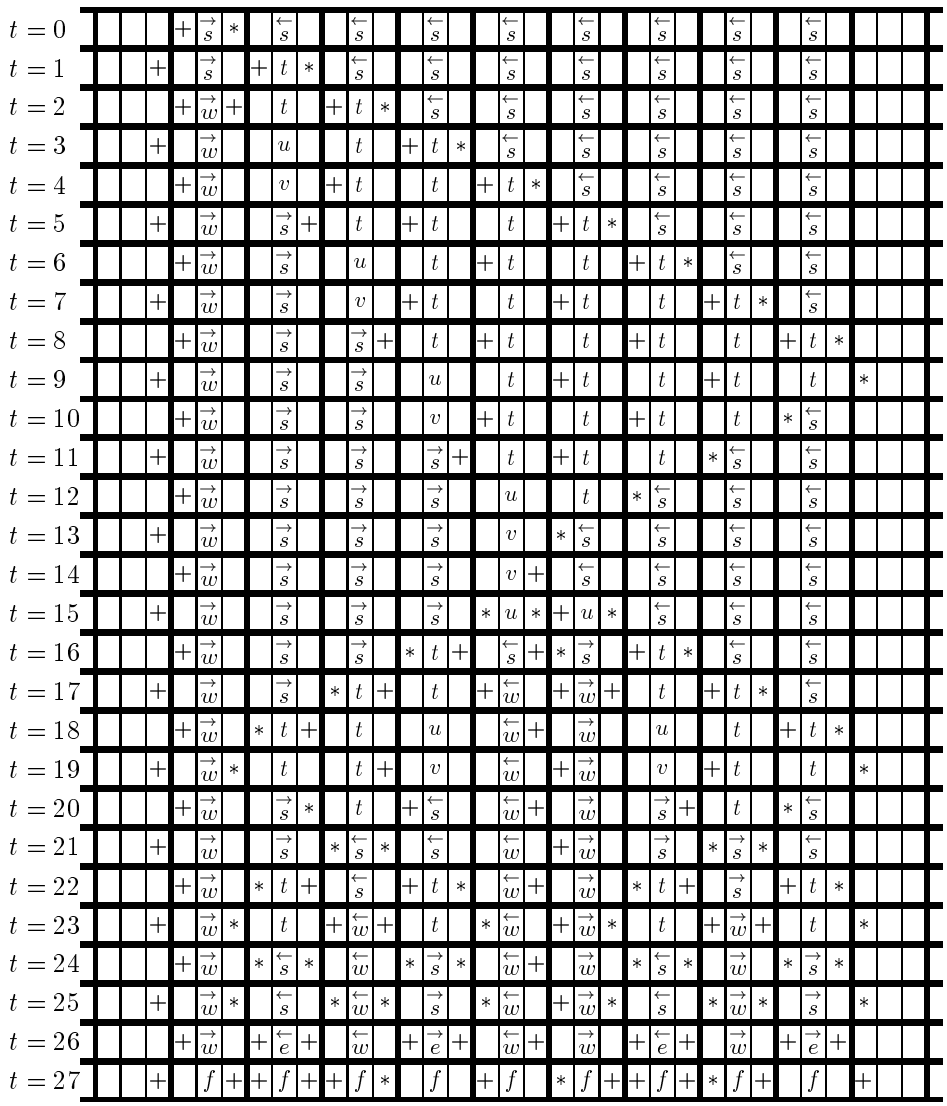

Figure 2: Synchronization of 9 cells by the 99-state reversible 1D PCA P.