Bilinear Cellular Automata

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Abstract

Bilinear cellular automata (CA) are those whose next state may be expressed as a bilinear form (an inner product) of the neighboring states. In this paper we show that, unlike linear CA, the bilinear CA over $\mathbf{Z}_p^p$ are π-universal, i.e. capable of simulating any CA of the same dimension, and hence also capable of simulating any (universal) Turing machine. Evidence is given that the bilinear CA over $\mathbf{Z}_m$, the integers modulo m, may be universal as well. (Although, like Conway's Game of Life, this appears to be difficult to establish, even for a prime number of states). Assuming that the bilinear CAs over $\mathbf{Z}_m$ are T-universal, we also give a fairly complete Wolfram type classification of the bilinear CA over $\mathbf{Z}_m$ according to properties of the coefficient matrix. As further evidence, we show bilinear automata are good models of discrete gravitational fields (monomial types), solitons, bounded growth, edge detection, and other applications.