Algebraic Aspects of One Dimensional Reversible Cellular Automata

Tim Boykett, Universität Linz.
tim@bruckner.stoch.uni-linz.ac.at

Pedersen's breakthrough work in [Ped92] has opened 1D CA to algebraic analysis. His work involves looking at CA as groupoids, examples of which permeate abstract algebra. A further development of his work gives a simple abstract algebraic characterisation of 1D reversible CA as $(2,2)$–algebra $(S, \bullet, \cdot)$ satisfying the axioms

$$(a \bullet b) \circ (b \bullet c) = b \tag{1}$$

$$(a \circ b) \bullet (b \circ c) = b \tag{2}$$

This structure is known as a *semicentral bigroupoid* as it generalises the *central groupoids* of Trevor Evans and Donald Knuth.

Elementary results, published in [Boy95] show an equivalent combinatoric formulation. These *rectangular structures* are easily seen to be equivalent to the tables of David Hillman in [Hil91]. They have a number of other interesting properties, however, relating to some questions in matrix theory and digraph theory.

Recent work has resulted in efficient combinatoric algorithms to exhaustively enumerate examples, reaching currently up to order 10. In this formulation, algebraic concepts of particle–like behaviour can be formulated, resulting in the discovery of significant amounts of such behaviour in the examples constructed.

A survey of these results will be presented, along with various interesting open questions.

References

[Boy95] Tim Boykett. Combinatorial construction of one dimensional reversible cellular automata. In G. Pilz, editor, *Contributions to General Algebra 9*, pages 81–90. Hölder–Pichler–Tempsky, Teubner, 1995.

[Hil91] David Hillman. The structure of reversible one–dimensional cellular automata. *Physica A*, 52:277–292, 1991.

[Ped92] John Pederson. Cellular automata as algebraic systems. *Complex Systems*, 6:237–250, 1992.