SHIFT-ADD CORRELATION PATTERNS OF LINEAR CELLULAR AUTOMATA ARE PSEUDO CELLULAR AUTOMATA.

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We consider linear cellular automata (CA) defined over a finite commutative ring $\mathcal{R}$ with multiplicative unity. *Shift-add correlation patterns* were introduced in [1],[2] as a means to detect hidden structures in coarse-graining invariant orbits of cellular automata with a linear rule in $\mathbb{Z}_p$.

Dual arrays of shift-add correlation patterns

Let $\bar{a}(r,c)\overset{\mathsf{def}}{=}(a(r,c))_{(r,c)\in\mathbb{N}\times\mathbb{Z}}$ be a two-dimensional CA-orbit obtained from the one-dimensional initial configuration $(\bar{a}(0,c))_{c\in\mathbb{Z}}$ through the linear recursion

$$a(r,c)=\sum_{j\in R_1}\rho(1,j)a(r-1,c-j)\,,$$

with R_1 a finite subset of $\mathbb{Z}$, and fixed coefficients $\rho(1,j)\in\mathcal{R}^*$ for all $j\in R_1$ ($\mathcal{R}^*$ is the set of nonzero elements in $\mathcal{R}$). Let (v,h) be a fixed element of $\mathbb{N}\times\mathbb{Z}$, and let α and β be fixed parameters in $\mathcal{R}^*$, then the (v,h)-*shift-add correlation pattern* $\bar{s}_{v,h}(r,c)$ is defined as the two-dimensional sequence $(s_{v,h}(r,c))_{(r,c)\in\mathbb{N}\times\mathbb{Z}}$, with

$$s_{v,h}(r,c)\overset{\mathsf{def}}{=}\alpha a(r,c)+\beta a(r+v,c+h).$$

(all operations are in $\mathcal{R}$ of course.)
The *array* $\bar{\mathcal{S}}(v,h)$ of shift-add (correlation) patterns is now defined as the two-dimensional sequence of patterns $(\bar{s}_{v,h}(r,c))_{(v,h)\in\mathbb{N}\times\mathbb{Z}}$.

In a similar way, with (r,c) a fixed element in $\mathbb{N}\times\mathbb{Z}$, we can consider the *dual* (r,c)-shift-add pattern as the two-dimensional sequence

$$\bar{s}^*_{r,c}(v,h)\overset{\mathsf{def}}{=}(s^*_{r,c}(v,h)\overset{\mathsf{def}}{=}\alpha a(r,c)+\beta a(r+v,c+h))_{(v,h)\in\mathbb{N}\times\mathbb{Z}}.$$

Also, the *array* of *dual* shift-add patterns $\bar{\mathcal{S}}^*(r,c)$ is defined as the two-dimensional *sequence of patterns* $\bar{s}^*_{r,c}(v,h))_{(r,c)\in\mathbb{N}\times\mathbb{Z}}$.

When represented graphically as arrays of patterns, both $\bar{\mathcal{S}}(v,h)$ and $\bar{\mathcal{S}}^*(r,c)$ can be seen as a dual pair of natural visualisations of the four-dimensional sequence

$$(s(v,h,r,c)\overset{\mathsf{def}}{=}\alpha a(r,c)+\beta a(r+v,c+h))_{(v,h,r,c)\in\mathbb{N}\times\mathbb{Z}\times\mathbb{N}\times\mathbb{Z}}.$$

Regular CA and associated pseudo-CA

A (one-dimensional) CA with the rule (in polynomial representation) $R(x)=\sum_{j\in R_1}\rho(1,j)x^j$, evolves according to the recursion

$$a(r,c)=\sum_{j\in R_1}\rho(1,j)a(r-1,c-j)\,,$$

starting with some initial state configuration $\bar{a}(0,c)$.
The corresponding *pseudo*-CA is defined as a CA-like structure that evolves according to the modified recursion

$$q(r,c)=\sum_{j\in R_1}\rho(1,j)q(r-1,c-j)+\gamma\theta(1)\,,$$

starting with the initial configuration $\bar{q}(0,c)=\bar{a}(0,c)$, and where γ is a fixed element of $\mathcal{R}^*$ and

$$\theta(1)\overset{\mathsf{def}}{=}1-\sum_{j\in R_1}\rho(1,j).$$

It is an interesting property to observe that, whereas a regular CA also satisfies the relationship

$$a(r,c)=\sum_{j\in R_v}\rho(v,j)a(r-v,c-j)$$

with $\rho(v,j)=\sum_{i\in R_1}\rho(1,j)\rho(v-1,c-j)$ and with $R_v\overset{\mathsf{def}}{=}\{j|\rho(v,j)\neq 0\}$ (for all $v\in\mathbb{N}$), the pseudo-CA satifies a corresponding relationship

$$q(r,c)=\sum_{j\in R_v}\rho(v,j)a(r-v,c-j)+\gamma\theta(v)\quad ,$$

where $\theta(v)\overset{\mathsf{def}}{=}1-\sum_{j\in R_v}\rho(v,j)$.

The relationship between shift-add correlation patterns and pseudo-CA

Consider a linear CA over $\mathcal{R}$, evolving under a given rule $R(x)$. For the associated shift-add correlations patterns and their duals, the following facts hold:

- $\bar{s}_{v,h}(r,c)$-patterns are regular CA-patterns evolving under the rule $R(x)$.
- the $\bar{\mathcal{S}}(v,h)$-*array* of $\bar{s}_{v,h}(r,c)$-patterns has the structure of a pseudo-CA orbit evolving under the rule $R(x)$.
- the dual $\bar{s}^*_{r,c}(v,h)$-patterns are pseudo-CA patterns evolving under the rule $R(x)$.
- the dual $\bar{\mathcal{S}}^*(r,c)$-*array* of $\bar{s}^*_{r,c}(v,h)$- patterns has the structure of a regular CA orbit evolving under the rule $R(x)$.

References

1 Barbé, A.,von Haeseler, F., Peitgen, H.-O., Skordev, G. [1995] "Coarse-graining invariant patterns of one-dimensional two-state linear cellular automata," *Int. J. Bifurcation and Chaos*, **5**(6), 1611-1631.

2 Barbé, A. [1996] "Coarse-graining invariant orbits of one-dimensional $\mathbb{Z}_p$ -linear cellular automata," to appear in *Int. J. Bifurcation and Chaos.*

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