Analysis of Discrete State Space Partitioned by the Attractors of the Dynamic Network Formation Game Model

19th International Workshop on Cellular Automata and Discrete Complex Systems
(AUTOMATA 2013)
September 17-19, 2013, Gießen

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Outline

- Background

  - *Boolean Networks* and *Random Boolean Networks*

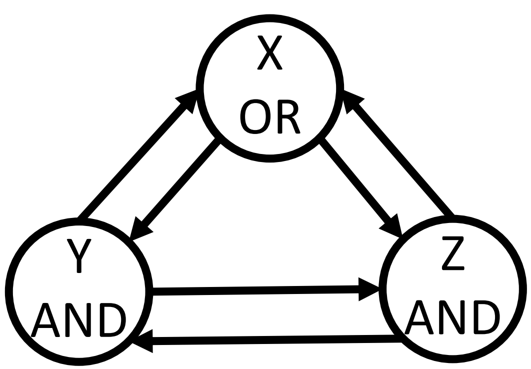
- *Dynamic Network Formation Game model*
- Complex behavior of this model similar to that of Random Boolean Networks
- Analysis of dynamics of this model in analogous way for analyzing Random Boolean Networks

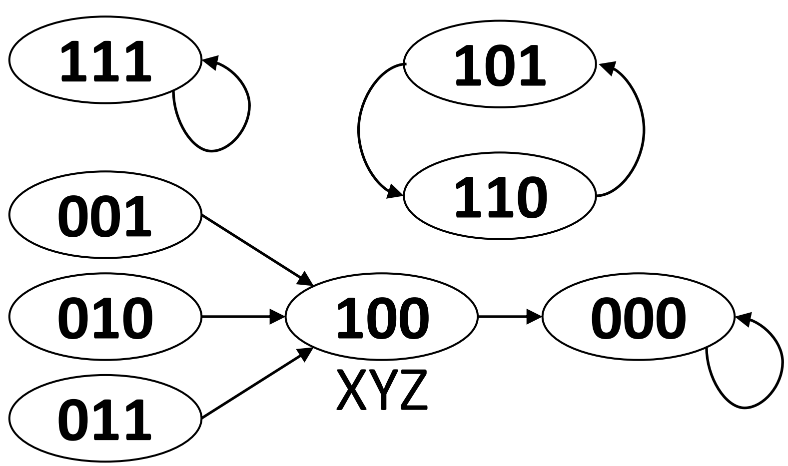
Background : Overview

- *Boolean Network (BN)* is
  - the system which consists of the set of Boolean variables which are determined in each discrete time step depending on other Boolean variables of the system.
- *Random Boolean Network (RBN)* is one of BN with N Boolean Variables that
  - K links among Boolean variables, and
  - Boolean functions determining the value of next step depending linked variables

  are randomly chosen.

ex.)





Background : Dynamics of RBNs

- Kauffman [Kauffman,1993] studied the behavior of RBNs and he classified RBNs into

  - *ordered phase*
  - *critical phase*
  - *chaotic phase*

  according to the number of K.

- RBN dynamics are analyzed by various measures such as

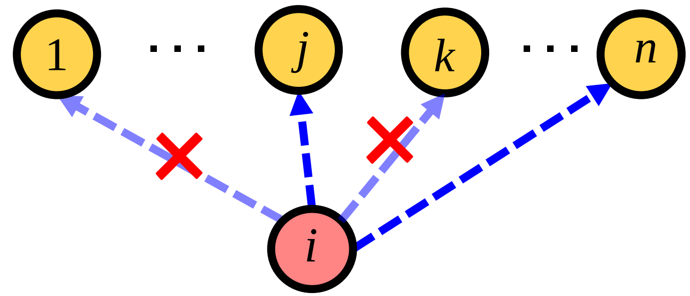
  - the number of attractors
  - the size of basin of attraction
  - the length of cycles
  - transient times
  - G-density
  - etc.

[Kauffman, 1993] S. Kauffman, *The Origins of Order: Self Organization and Selection in Evolution,* Oxford University Press, Incorporated, 1993.

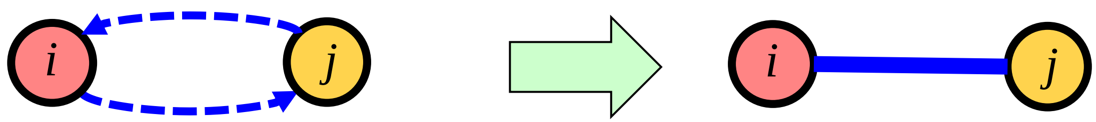
Dynamic Network Formation Game Model [Imai et al.,2010]

- (Static) Network Formation Game (*NWFG*)
  - A non-cooperative game which is known in the field of game theory
  - **Network formation by selfish and distributed multiple agents**
- *Acceptable links* and *Pairwise stability*
- Dynamic NWFG model
  - This model represents **network growth** by introducing dynamicity to the static NWFG
  - **Power-law degree distributions** known as ***complex network***

(Static) network formation game [Jackson et al., 1996]



- **Player**
  - Node (=vertex)
- **Strategy**
  - Intentions of forming links(=edges).
  - Each player declare that he/she **wishes to form a link** to each other player.
- **Outcome**



  - If and only if **both of two players** wish to form a link between them, then it is actually formed.
  - It determines an overall outcome topology(=graph).

[Jackson et al., 1996] M.O. Jackson and A. Wolinsky, "A strategic model of social and economic networks," Journal of Economic Theory 71(1) pp. 44-74, 1996.

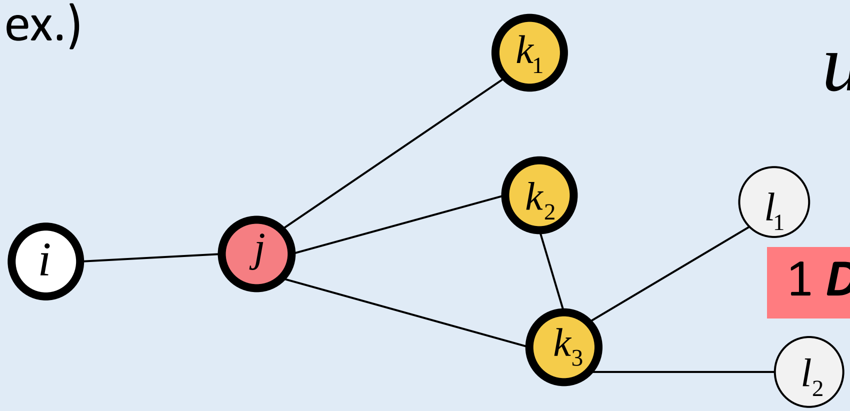
Payoff Function

$$u_i(g) = \sum_{j \in \{j | ij \in g\}} (\delta - c_{ij}) + \sum_{j \in \{j | ij \notin g\}} \delta^{d_{ij}}$$

δ : decay parameter, $0 < \delta < 1$
c_{ij} : cost parameter which are randomly sampled from $(0, R]$
d_{ij} : distance between i and j

- **Direct connection** : **Maximal benefit** and **link cost**
- **Indirect connection** : **Decayed benefit** and **no link cost**

ex.)



$$u_i = \underbrace{(\delta - c_{ij})}_{\text{1 Direct connection}} + \underbrace{3\delta^2}_{\text{3 indirect connections with distance 2.}} + \underbrace{2\delta^3}_{\text{2 indirect connections with distance 3.}}$$

1 ***Direct connection***

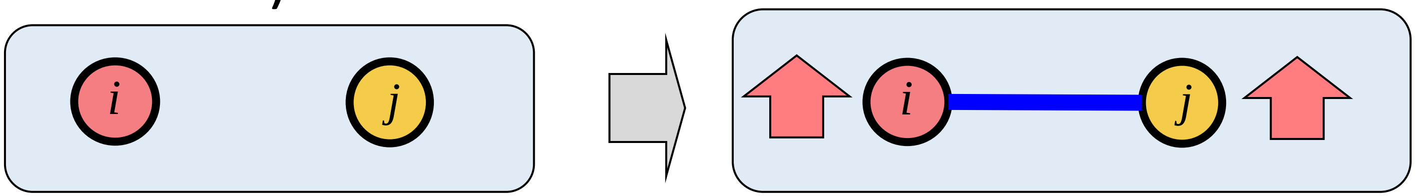
2 ***indirect connections*** with distance 3.

3 ***indirect connections*** with distance 2.

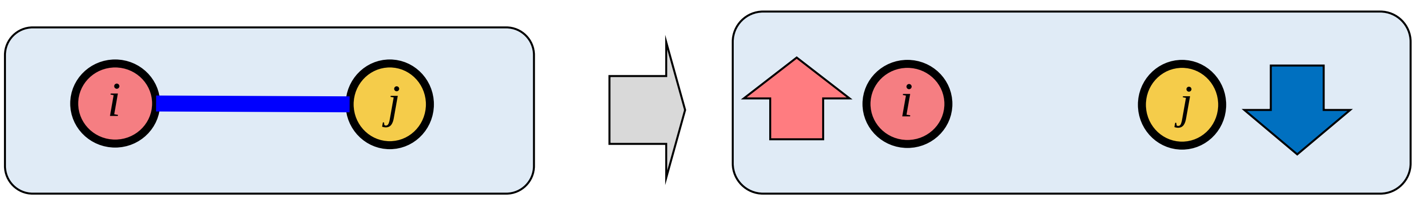
Cost for information and **decayed information value**

Acceptable Links and *Pairwise stability*

- *Acceptable link*
  - For adding : Both of two involved players' payoffs increase (or one remain)



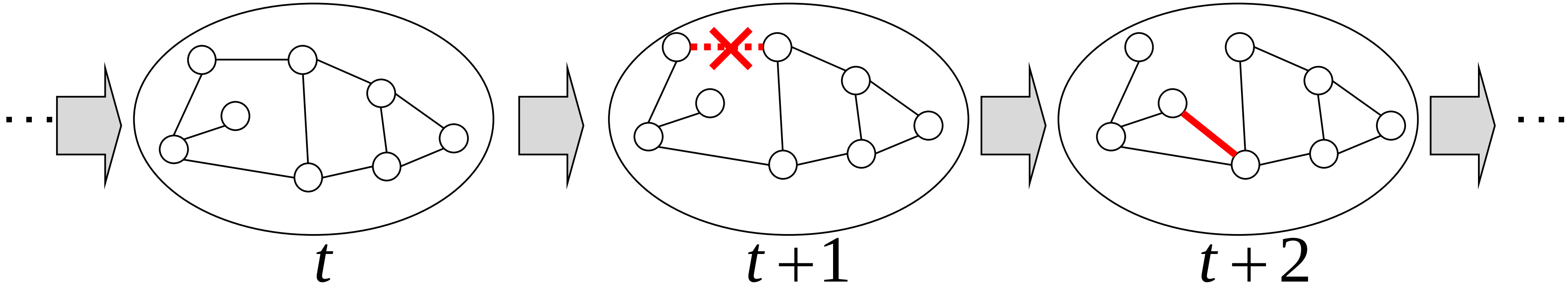
  - For removing : At least one involved players' payoff increases (or remain)



- *Pairwise stable* topology
  - ***No acceptable links*** in the topology
  - A solution concept for NWFG

Dynamic NWFG model [Imai et al.,2010]

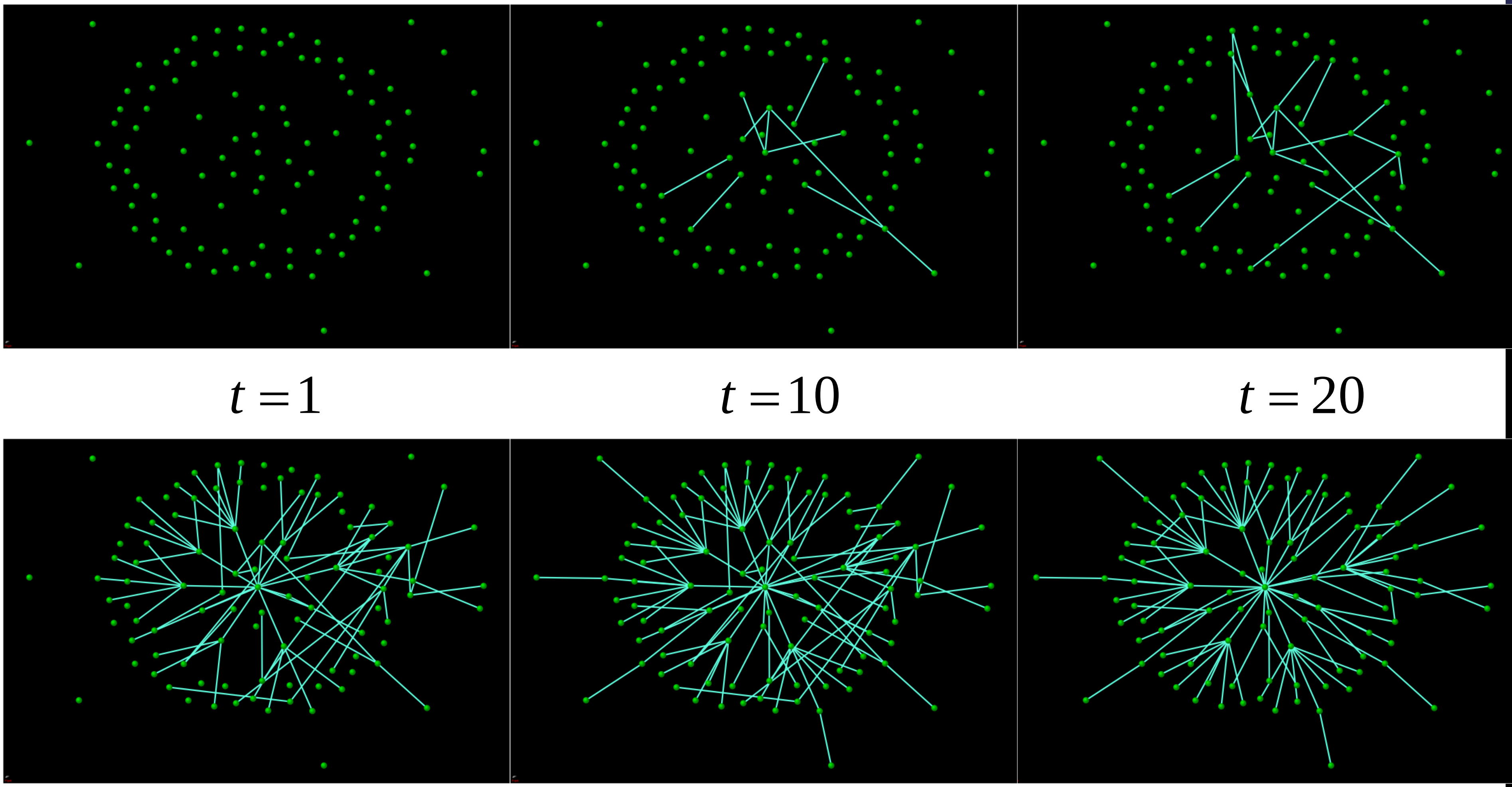
- A time series of the static NWFG.
- **At most only one acceptable link** changes at each time step t.



- **The most payoff improving link** among all acceptable links can change at each time step t.
- This process continues to converge to any pairwise stable attractors or cycle attractors.

[Imai et al., 2010] T. Imai, A. Tanaka, "A Game Theoretic Model for AS Topology Formation with the Scale-Free Property," IEICE TRANSACTIONS on Information and Systems, Vol.E93-D, No.11, pp.3051-3058, 2010.

Topology transition by dynamic NWFG Model



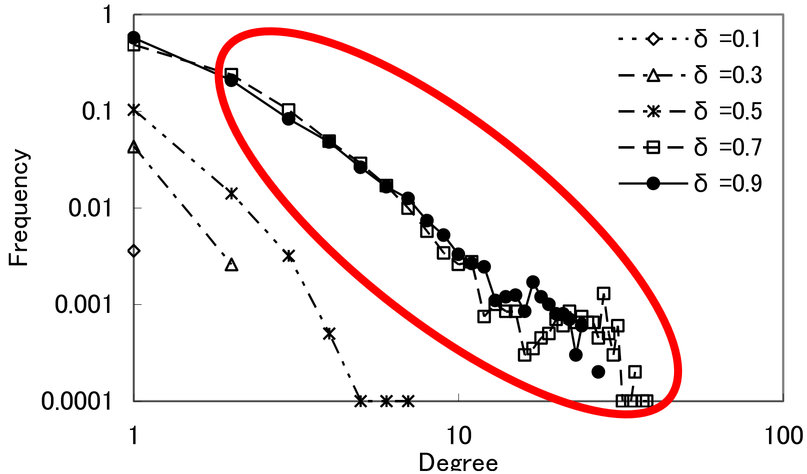
$t=1$ $t=10$ $t=20$

$t=70$ $t=120$ $t=145$ (Pairwise Stable)

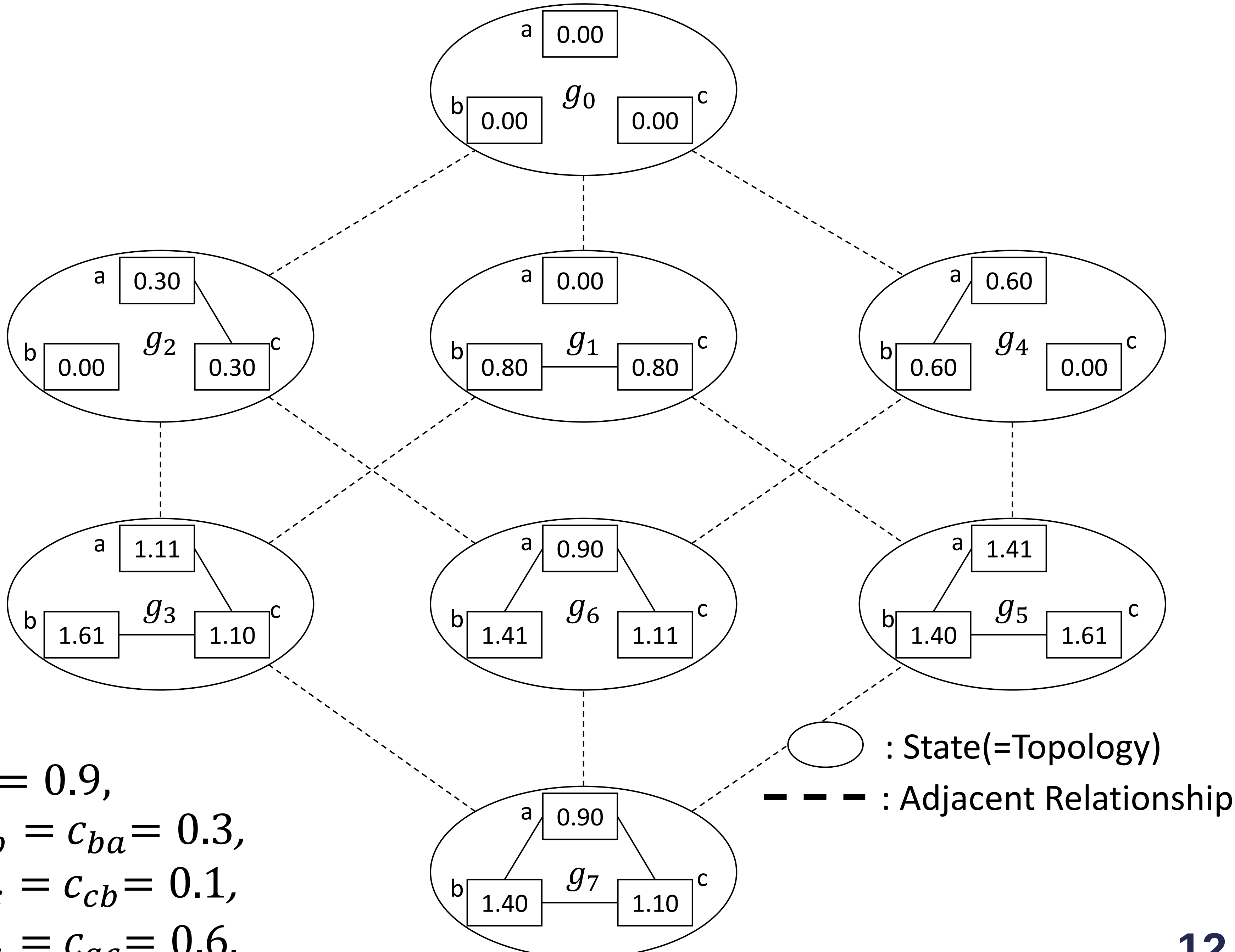
(100nodes, $\delta=0.9$, $c \in (0.0,20.0]$)

Properties of dynamic NWFG model

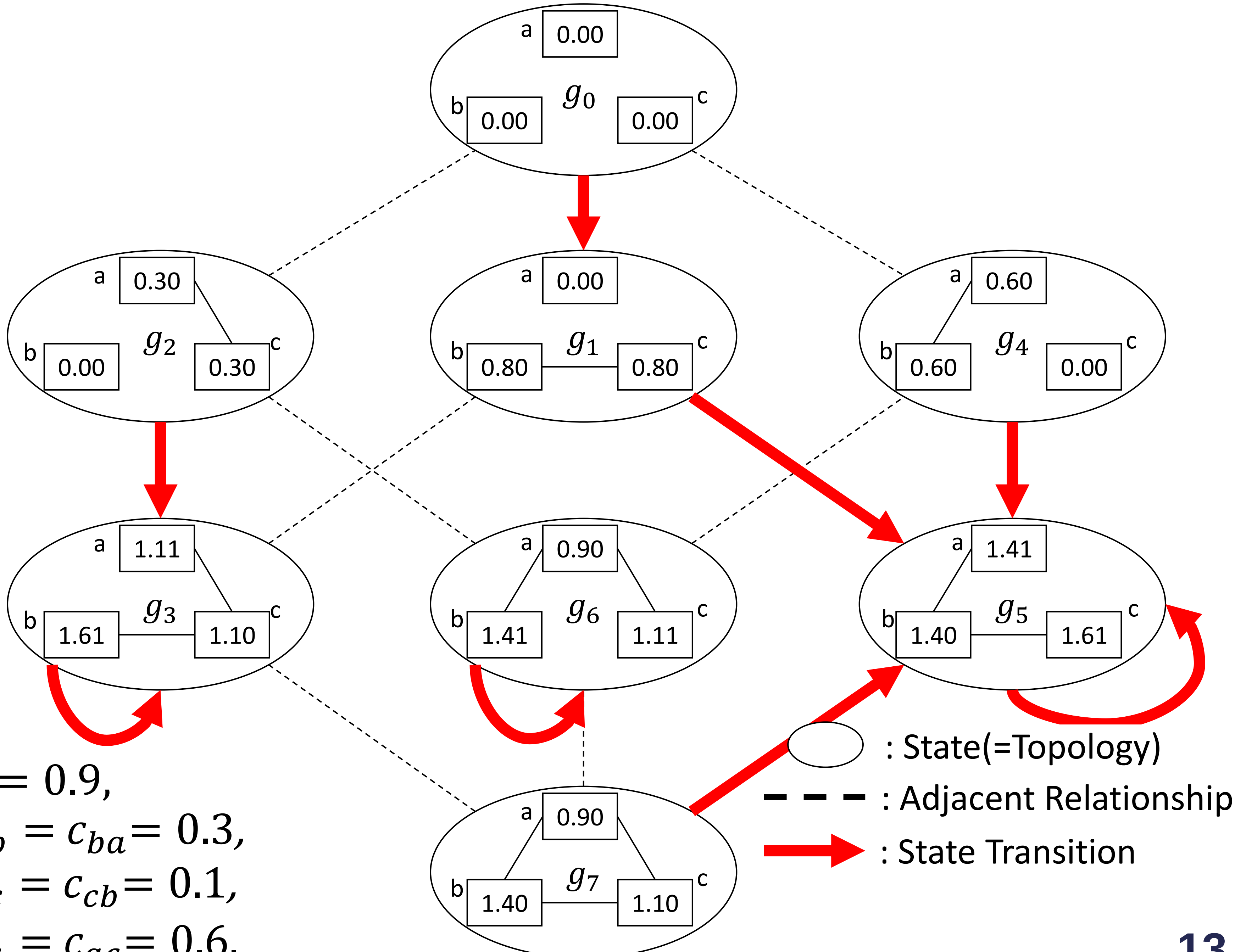
- **Deterministic** state transition process depending only on the current state
- **Point Attractors** and **Cycle Attractors**
- **All Point Attractors are *Pairwise Stable* topology** which is the solution concept of static NWFG.
- This model tends to emerge ***scale-free topology*** from initial empty topology in some settings.



Degree distribution of emerged topologies by dynamic NWFG model. [Imai et al.,2010] ($\delta = 0.9, R = 20.0$, with transfer)



$$\delta = 0.9,$$
$$c_{ab} = c_{ba} = 0.3,$$
$$c_{bc} = c_{cb} = 0.1,$$
$$c_{ca} = c_{ac} = 0.6.$$



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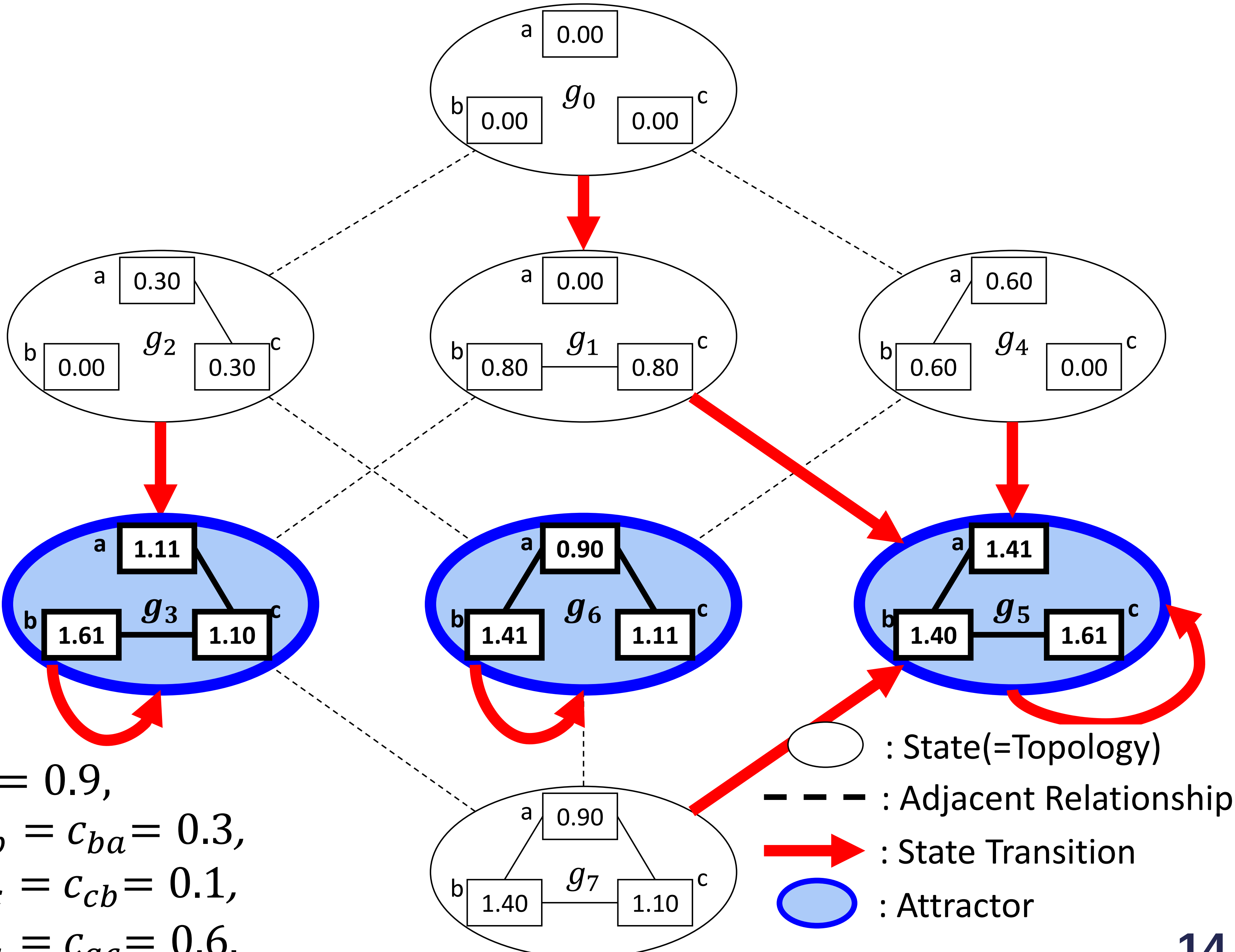

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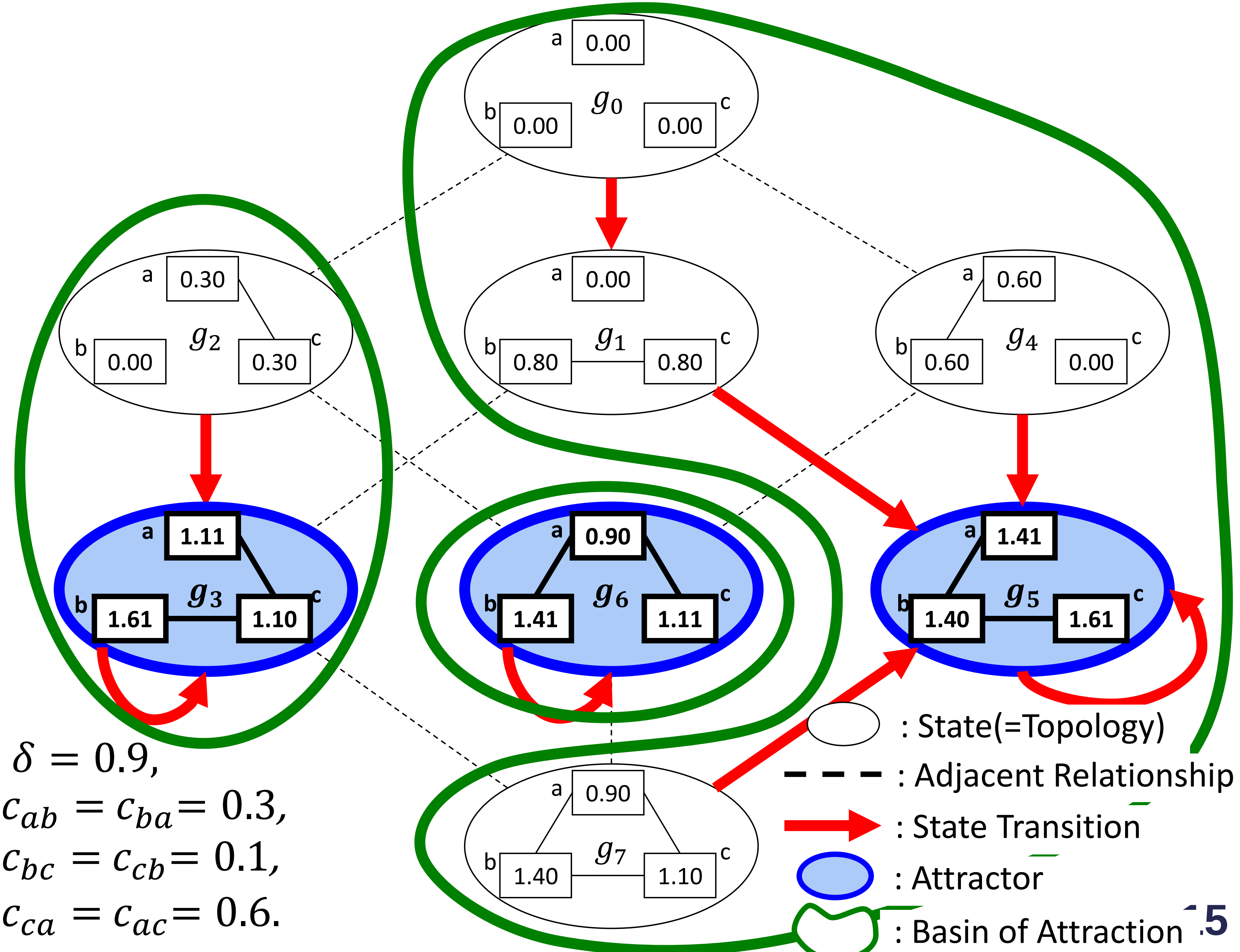

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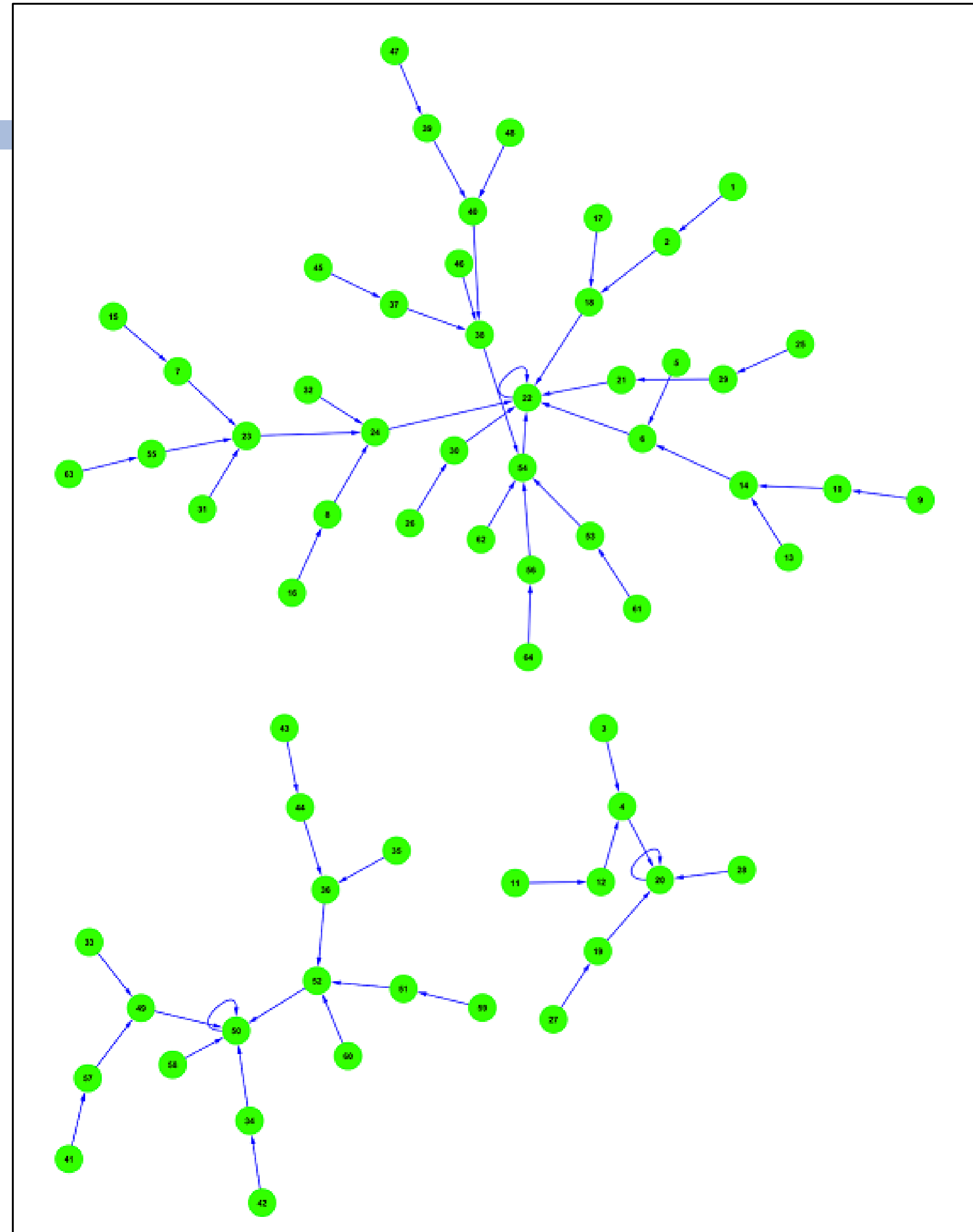
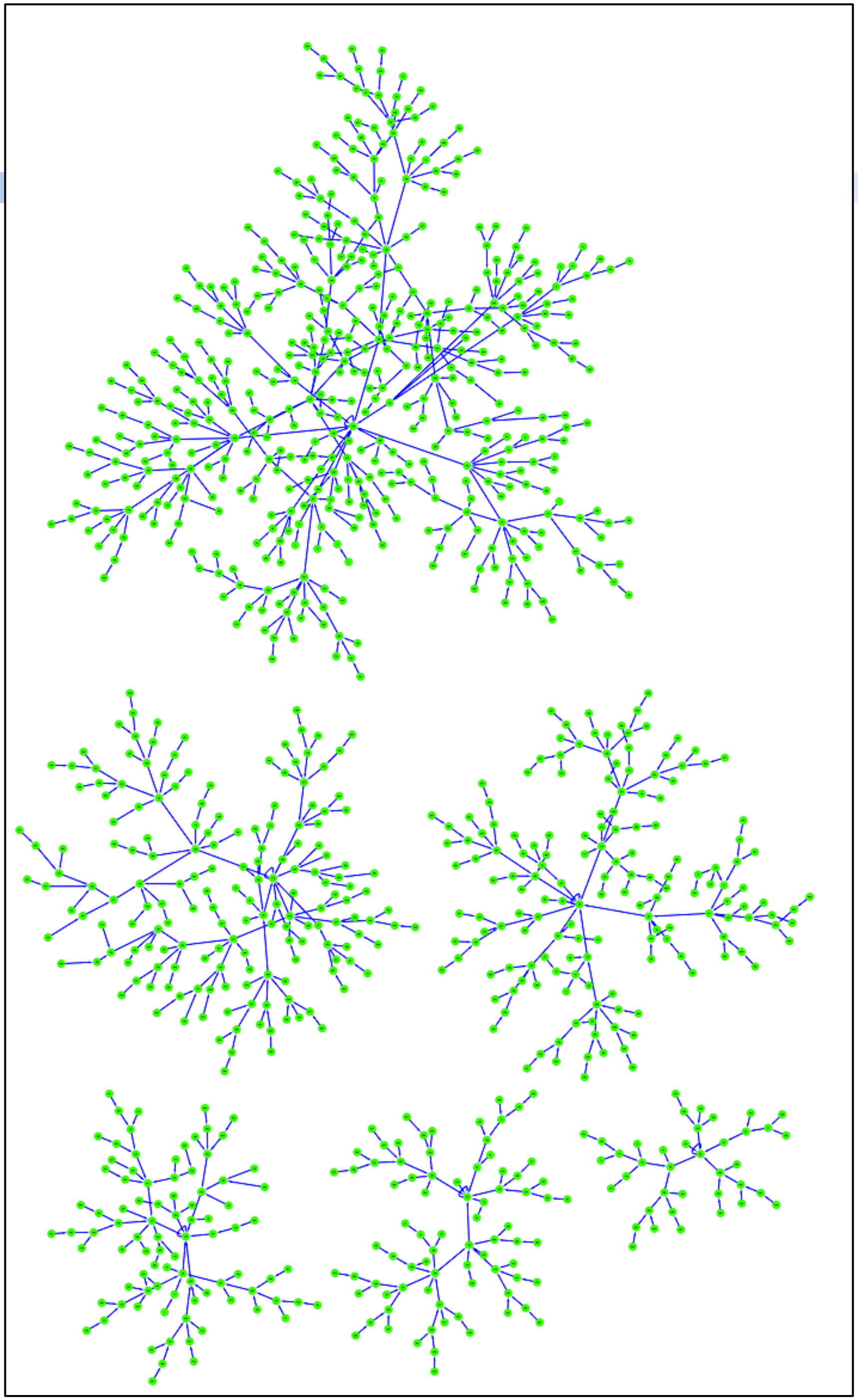
Example 1:
# of nodes = 4

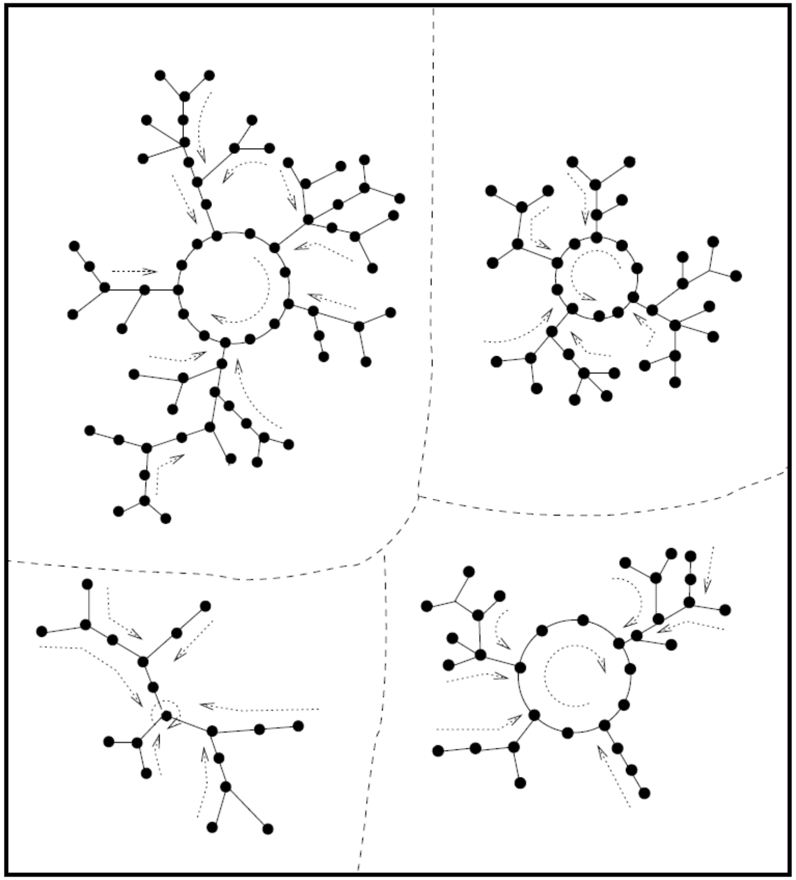
- $n = 4$
- (# of states) $= 2^{\binom{n}{2}} = 2^6 = 64$
- $\delta = 0.9$
- $R = 2.0$



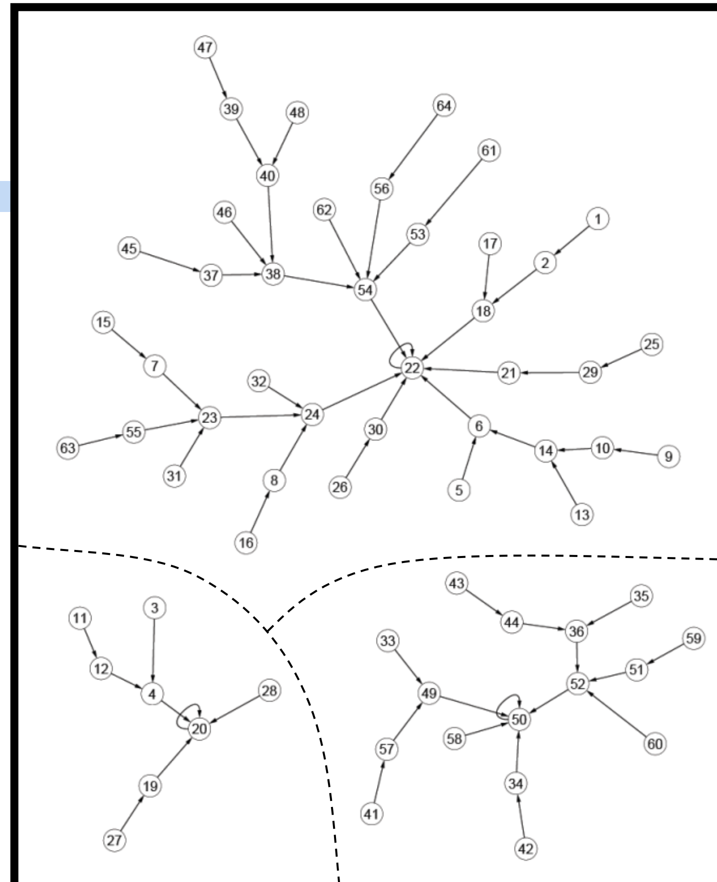
Example 2:
of nodes = 5

- $n = 5$
- (# of states) $= 2^{\binom{n}{2}} = 2^{10} = 1024$
- $\delta = 0.9$
- $R = 2.0$





a) RBN [Kadanoff et al., 2002]



b) Dynamic NWFG model

- Dynamic NWFG model **is also a kind of Boolean Networks**.
- **Complex state space is similar to that of RBNs.**
- **Analysis measures for RBN can also be applied to** analysis of the dynamic NWFG model.

[Kadanoff et al., 2002] L. Kadanoff et al., "Boolean dynamics with random couplings," *eprint arXiv:nlin/020462*, Apr. 2002.

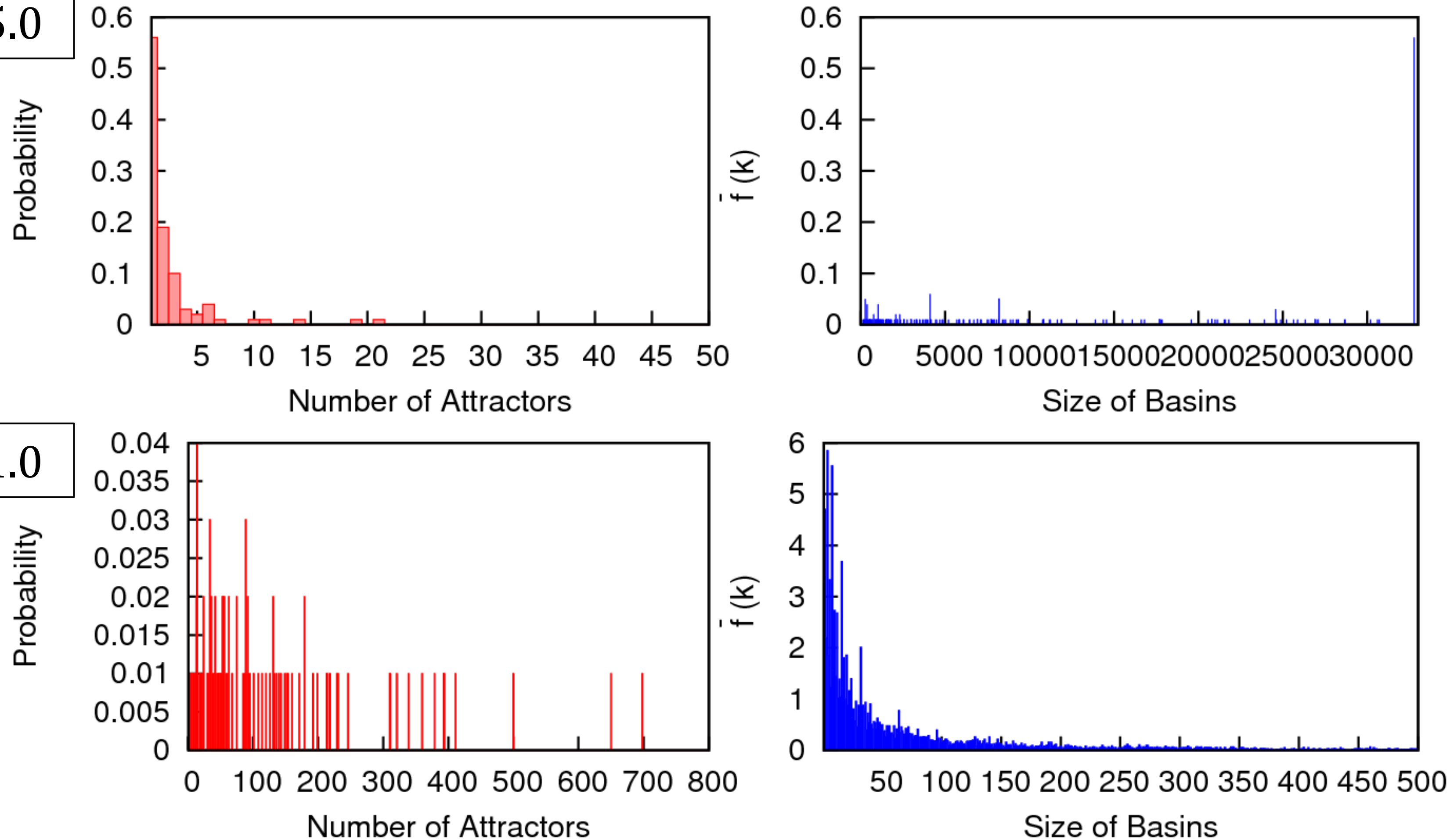
Investigation of partitioned state space

- The state space structures are specified by
  - decay parameter δ
  - randomly sampled cost parameters $c_{ij} \in (0, R]$.
- We investigated **100 patterns of partitioned state space** made by random parameters by observing
  - **The number of attractors**
  - **Size of basins of attraction**
- Settings :
  - # of nodes = 6
    - size of whole state space = 32,768
  - δ is fixed to 0.9,
  - $R = 1.0$ and $R = 5.0$

Properties of partitioned state space

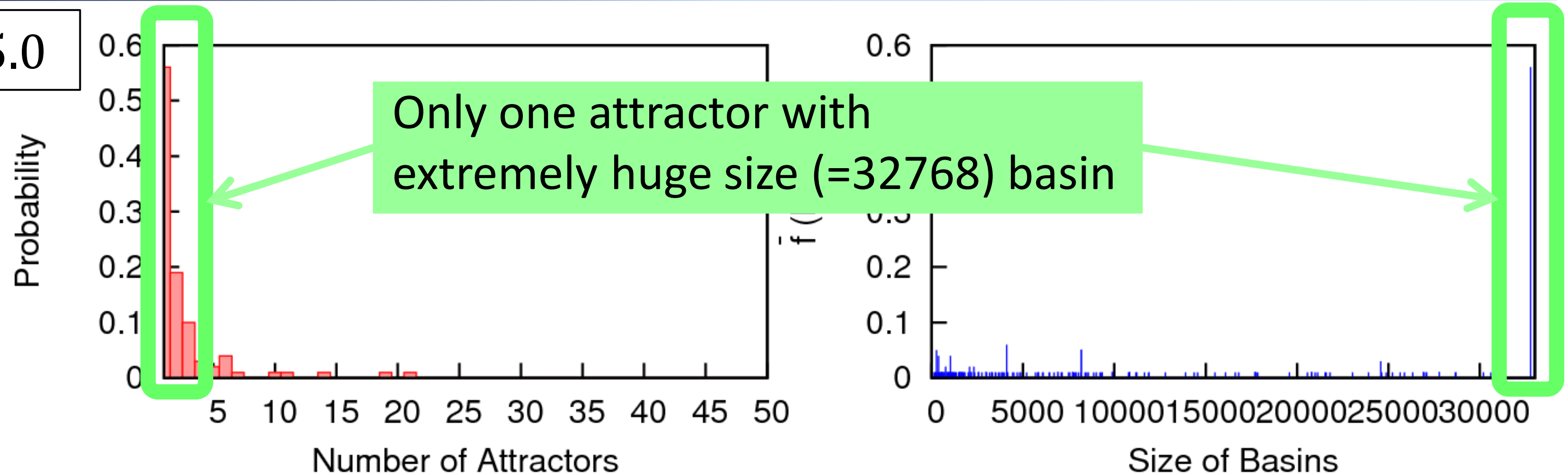
$R = 5.0$

$R = 1.0$

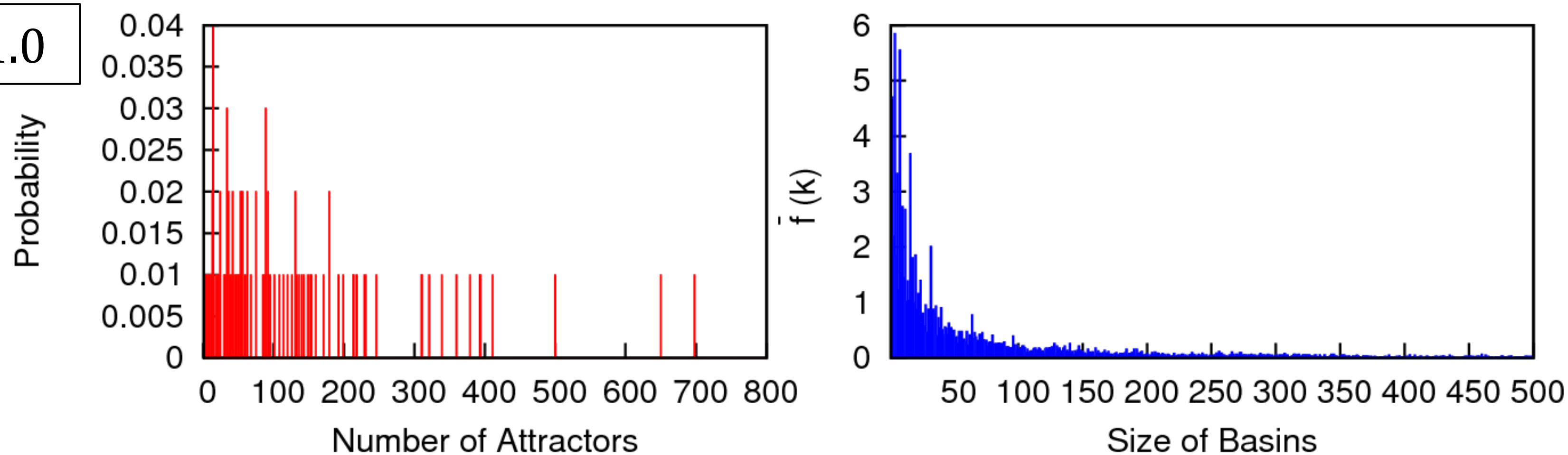


Properties of partitioned state space

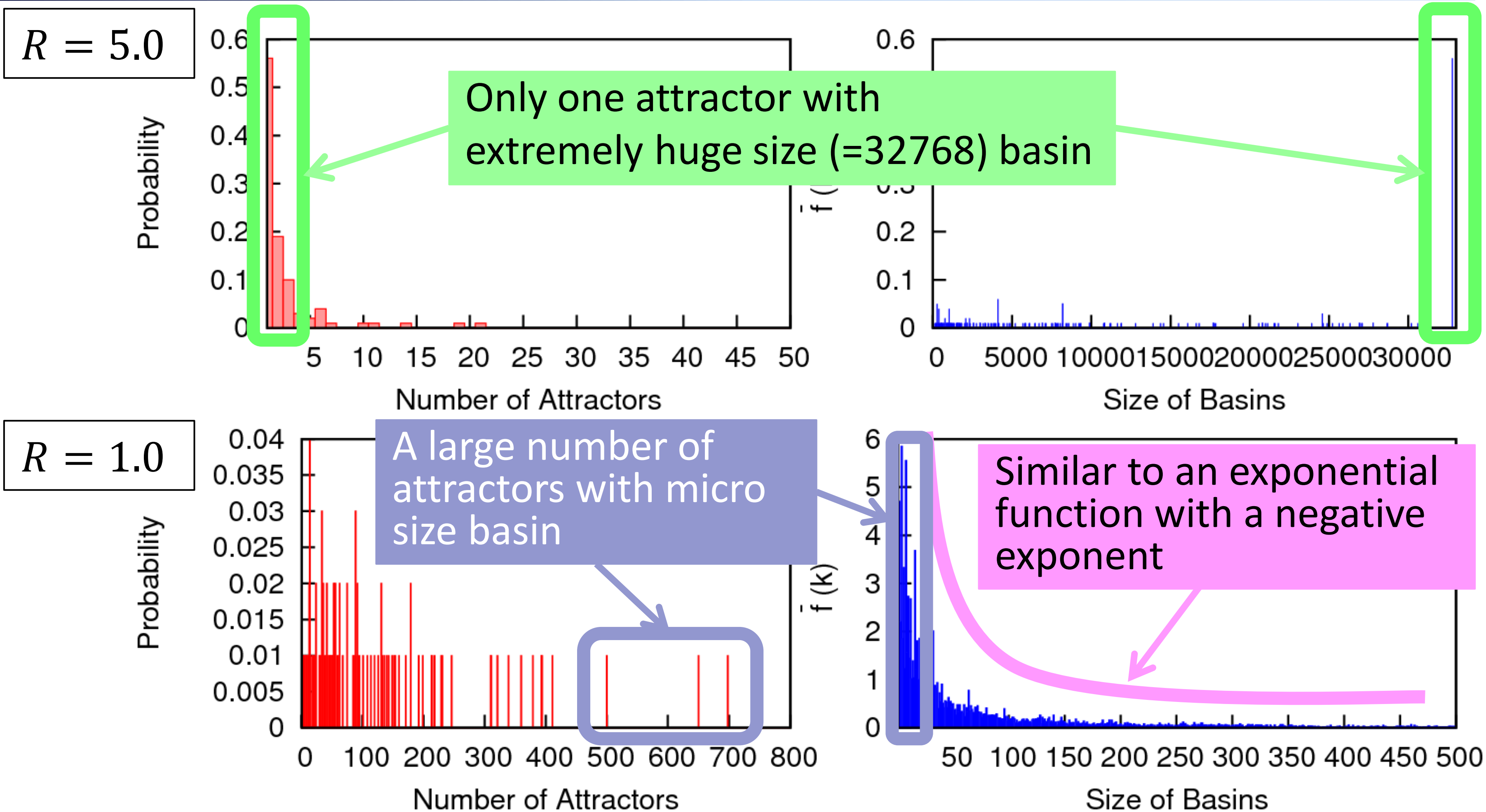
$R = 5.0$



$R = 1.0$



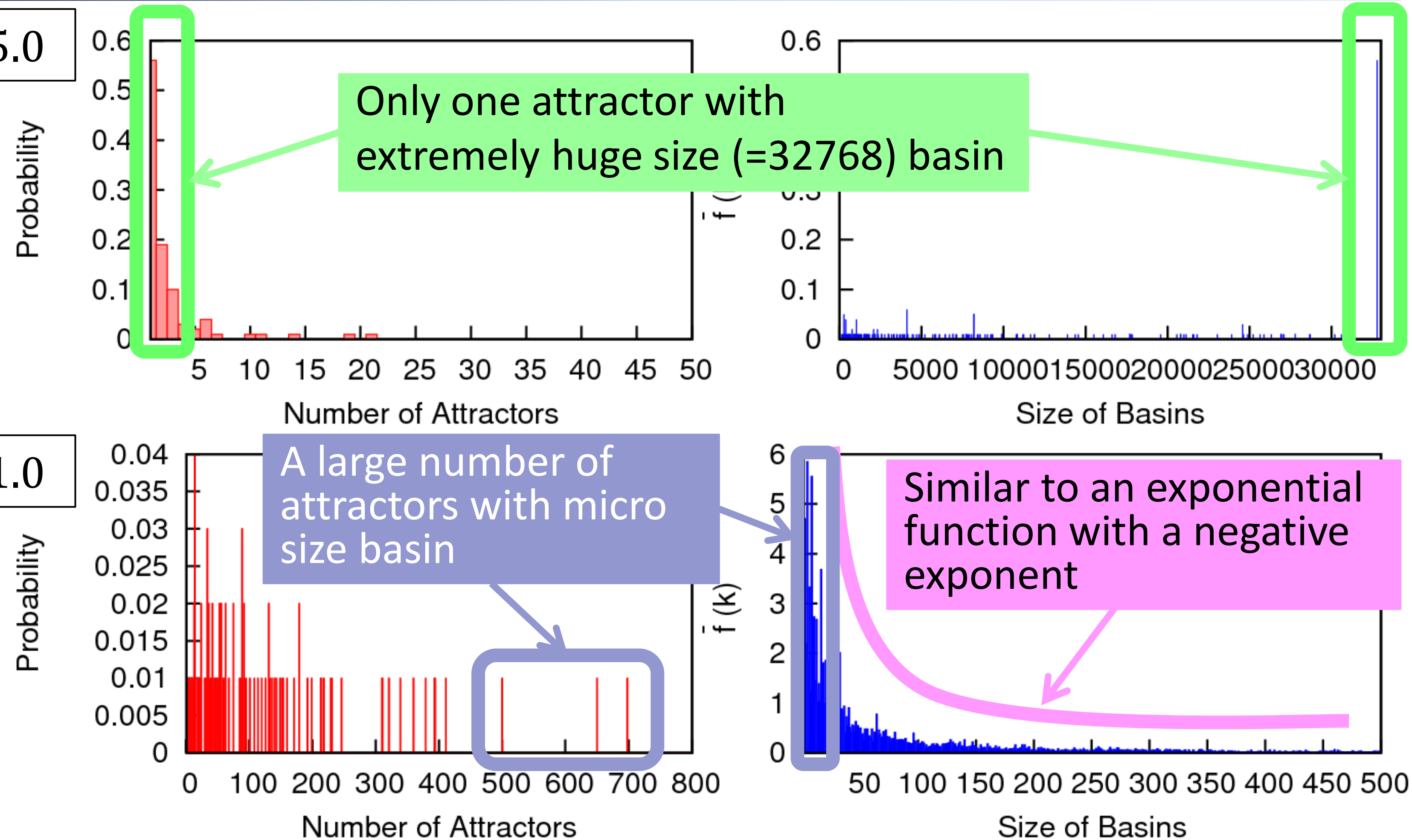
Properties of partitioned state space



Properties of partitioned state space

$R = 5.0$

$R = 1.0$



In the cases of some parameter sets, whole state space is partitioned into basins of ***the diverse number and diverse size.***

Evaluation of validity of Monte Carlo method

- We can do **exhaustive investigations** over whole topology sets, **in the case of small nodes.**
- Truthfully, we want to know about dynamics of topologies **in the case of larger nodes**.

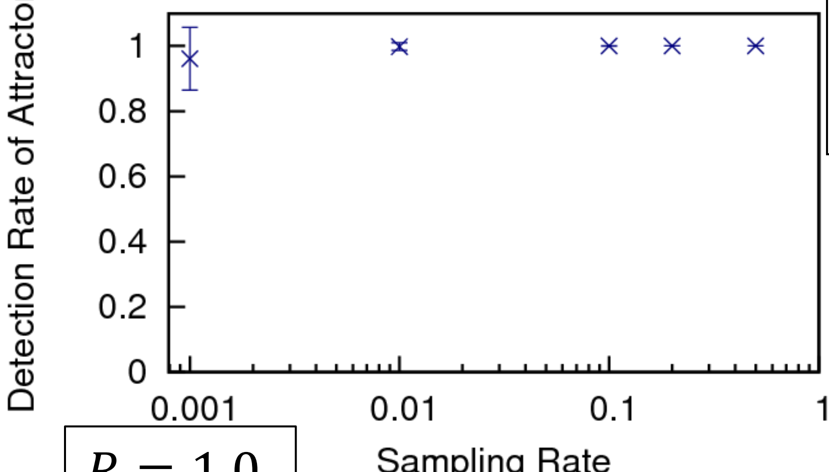
- **Monte Carlo method** is used to estimate properties of state space by little number of random samples.
- Is the Monte Carlo method **sufficiently effective for property estimation** of state space of dynamic NWFG model?

Results

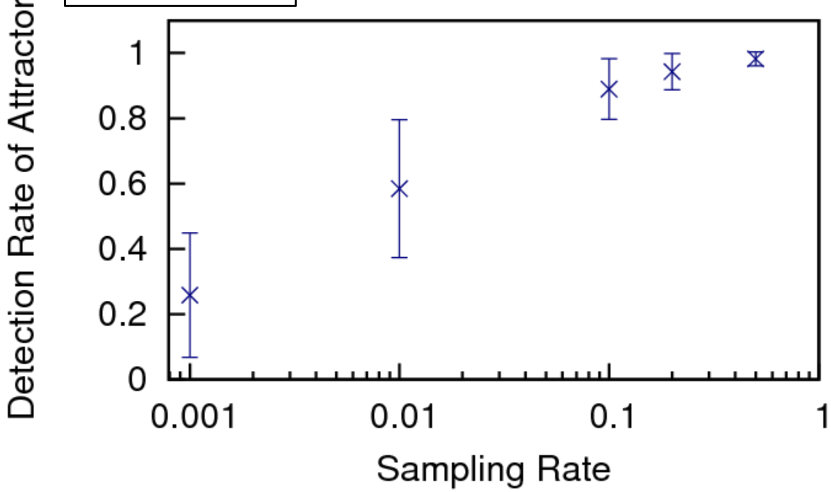
Attractor Detection Rate

$$= \frac{\text{(\# of actually detected attractors)}}{\text{(Total \# of attractors)}}$$

$R = 5.0$



$R = 1.0$



- Sampling rate :
  - 0.001, 0.01, 0.01, 0.2 and 0.5

- $R = 5.0$
  - Less diversity
  - **Only a small number of samples is needed** to detect all attractors certainly.
- $R = 1.0$
  - More diversity
  - **A large number of samples is needed** for certain detection.

Monte Carlo approach may not sufficiently be effective because it need **much samples for seriously complex state space.**

Summary

- Introduction of dynamic NWFG model
- The analysis measures for RBN can also be applied to analysis of the dynamic NWFG model.
- The partitioned state space of the dynamic NWFG model is highly diverse in some settings.
- Is Monte Carlo method sufficiently efficient?
  - Good in the case that partitioned state space is not so diverse,
  - Not so good in the case that it is seriously diverse.
- Future Work
  - We still stand on the entrance of Discrete Dynamic System analysis.
    - › We need to compare the dynamic NWFG model more precisely to other Boolean Network.
  - Classification of the dynamic NWFGs by parameters.

APPENDIX

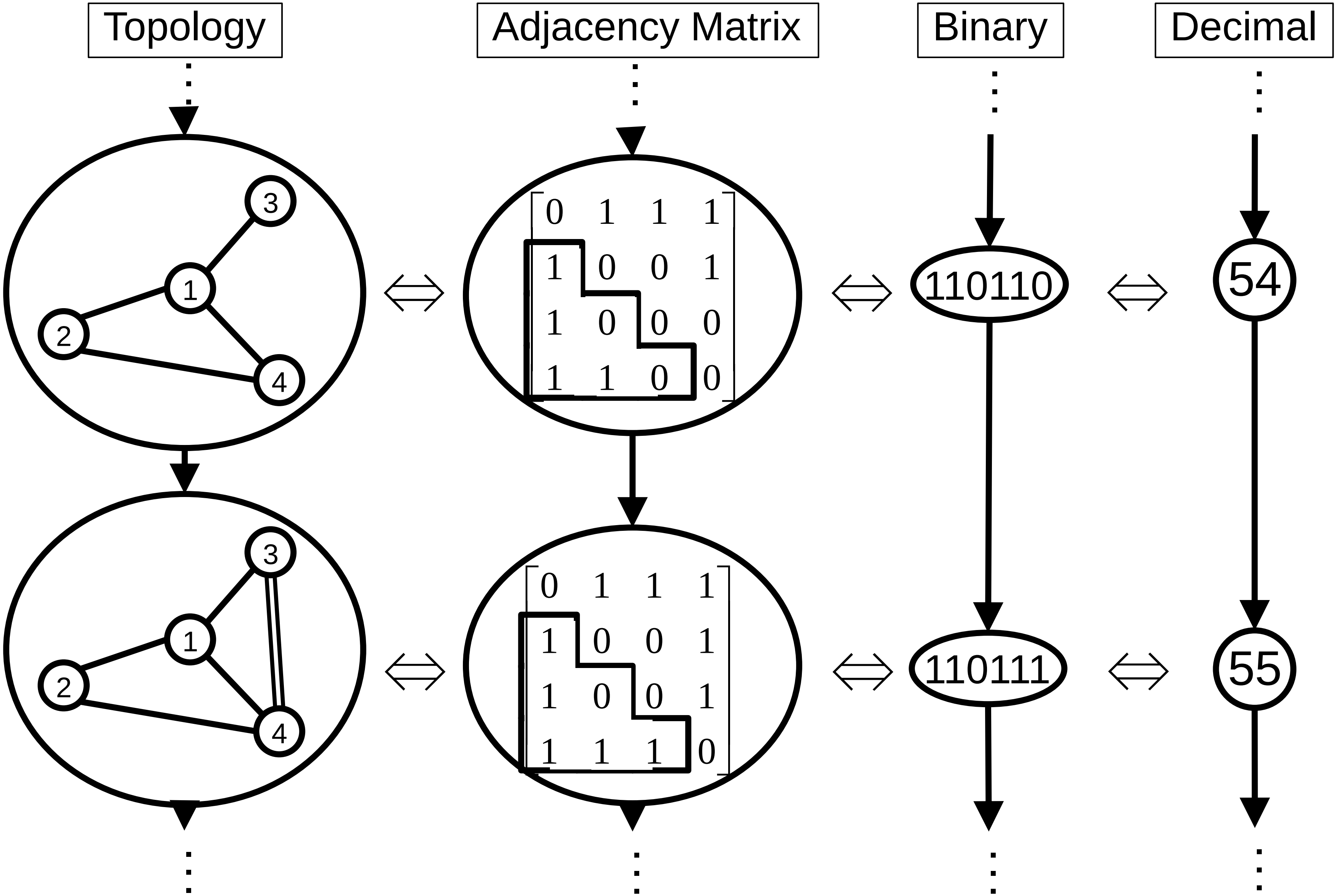
Position of our research

- Cross point of
  - *Complex Networks,*
  - *Game Theory* and
  - *Computational Science*
- *Complex Networks*
  - How complex networks are generated?
  - *"Why"* complex networks are generated?
- *Game Theory* and *Computational Science*
  - Computational/Algorithmic Game Theory
    - › Limited capability of
      - observation of the situation
      - optimization of its own strategy
    - › Refinement of solution concepts
    - › Path dependency

Exponential increase of the size of state space

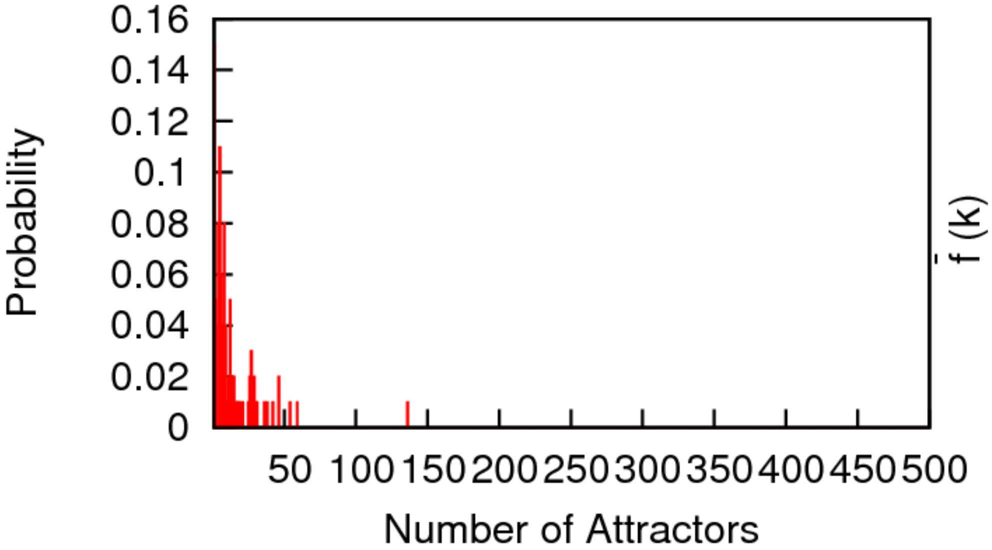
Table 6.1: Explosive Increase of the Size of State Space by Increase of the Number of Node

# of nodes	${}_nC_2$	$2^{{}_nC_2}$ =(the Size of State Space)
0	0	1
1	0	1
2	1	2
3	3	8
4	6	64
5	10	1,024
6	15	32,768
7	21	2,097,152
8	28	268,435,456
9	36	68,719,476,736
10	45	35,184,372,088,832
11	55	36,028,797,018,963,968
12	66	73,786,976,294,838,206,464
13	78	302,231,454,903,657,293,676,544
14	91	2,475,880,078,570,760,549,798,248,448
15	105	40,564,819,207,303,340,847,894,502,572,032
16	120	1,329,227,995,784,915,872,903,807,060,280,344,576
17	136	87,112,285,931,760,246,646,623,899,502,532,662,132,736
18	153	11,417,981,541,647,679,048,466,287,755,595,961,091,061,972,992

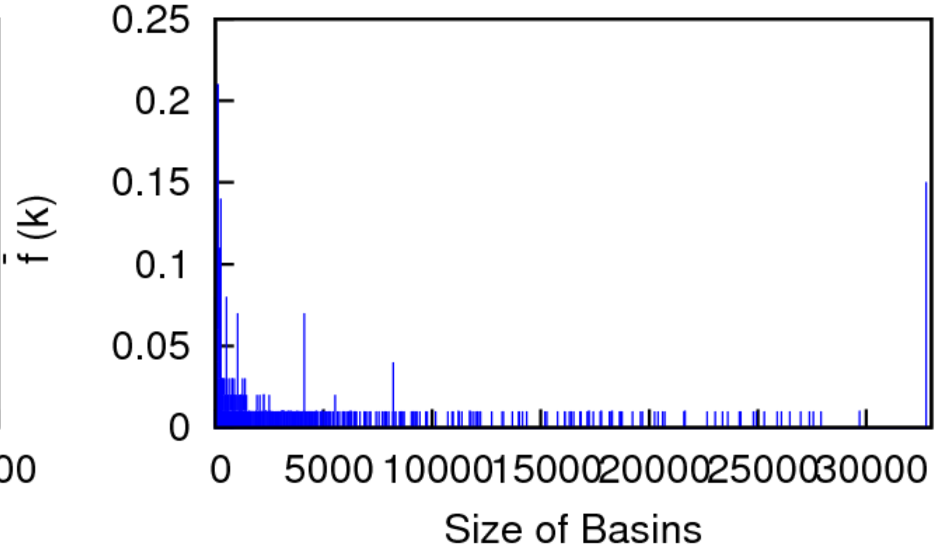


- The dynamic NWFG model (and topology transition models in general) give an additional explanation to a state which has only been a bit sequence.

$\delta = 0.9, R = 3.0$

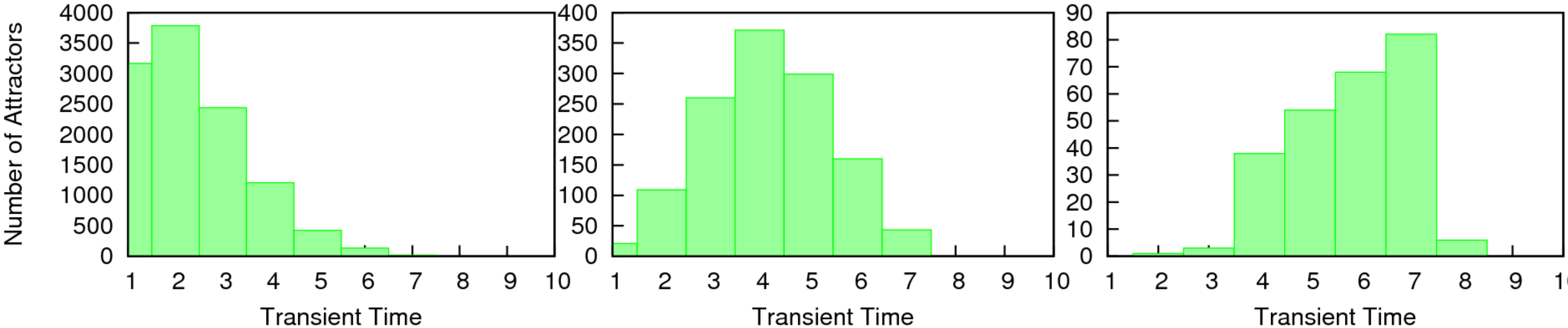


Probability
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
50
100
150
200
250
300
350
400
450
500
Number of Attractors



f̄ (k)
0.25
0.2
0.15
0.1
0.05
0
0
5000
10000
15000
20000
25000
30000
Size of Basins

Transient Time

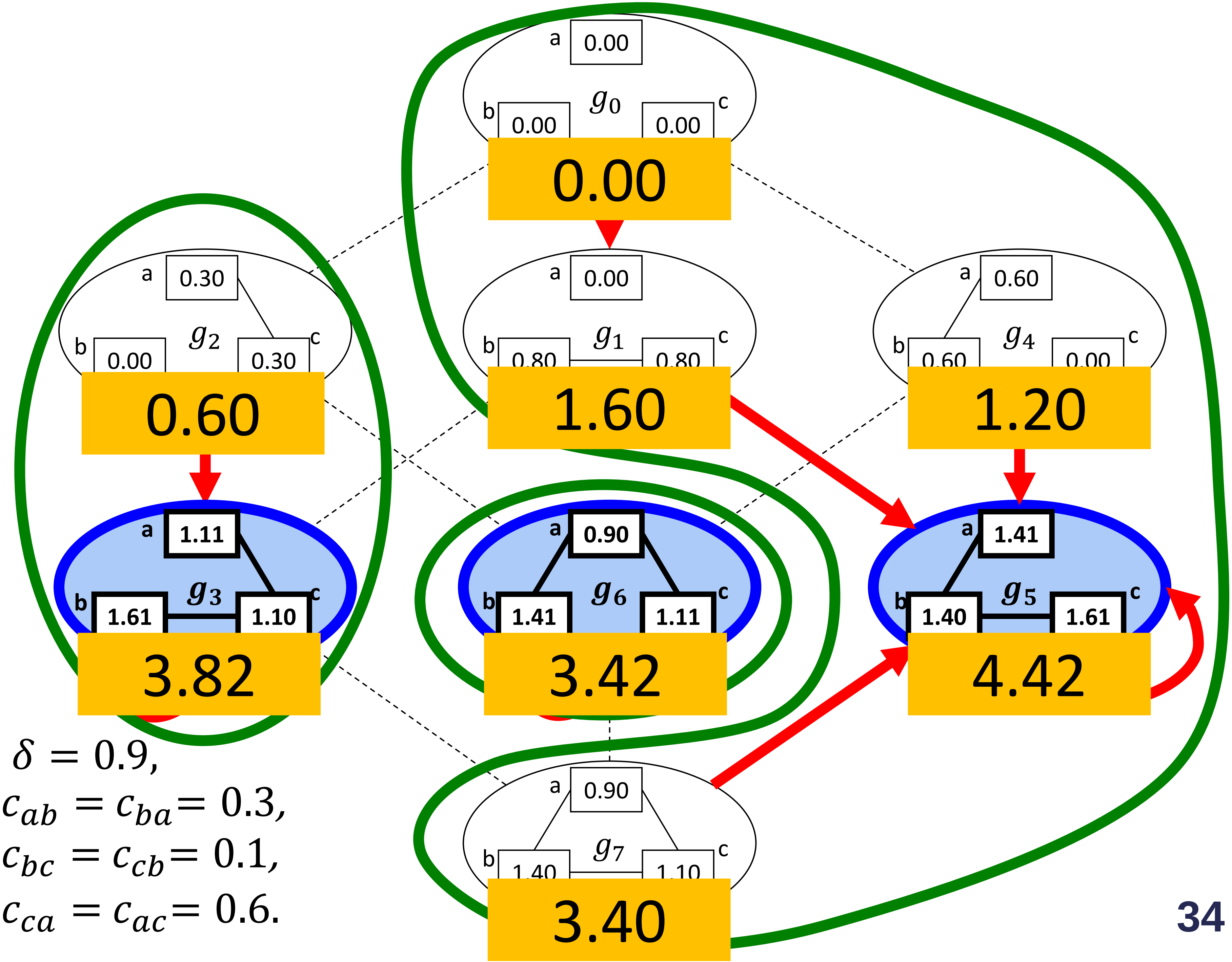


$\delta = 0.9, R = 1.0$ $\delta = 0.9, R = 3.0$ $\delta = 0.9, R = 5.0$

- Transient time is
- ランダムに100個生成されたパターンで，ある平均Transient Timeを持つアトラクタがいくつ存在するか，を示したグラフ.

Q&A

- 対称性がなぜ破れているのか，説明する必要があるかもしれない．
- 現在の状態が望ましくないアトラクタをもたらす場合，コストを変化させることによって状態空間の性質を変化させて，より望ましいアトラクタに導くことができるかもしれない．



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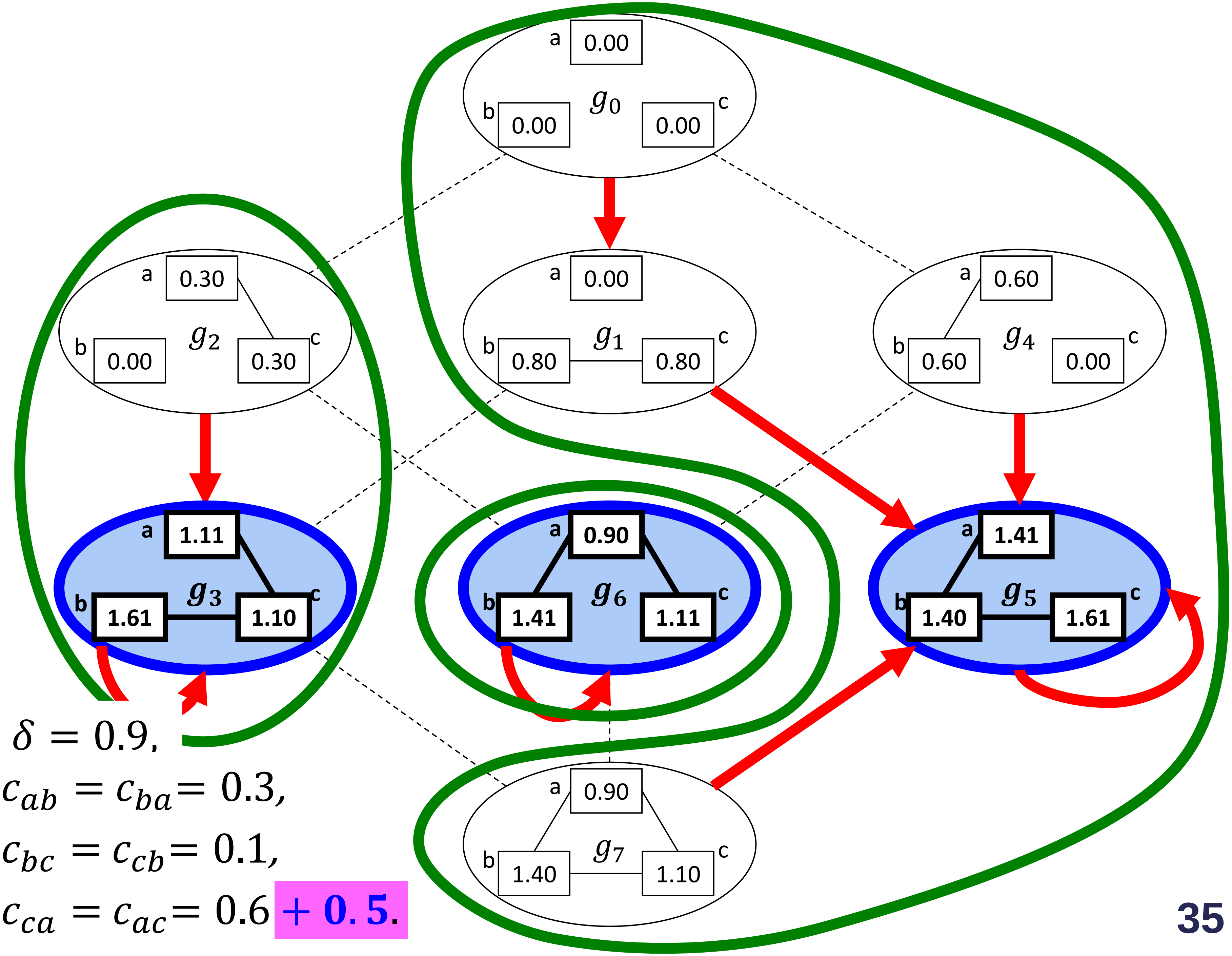
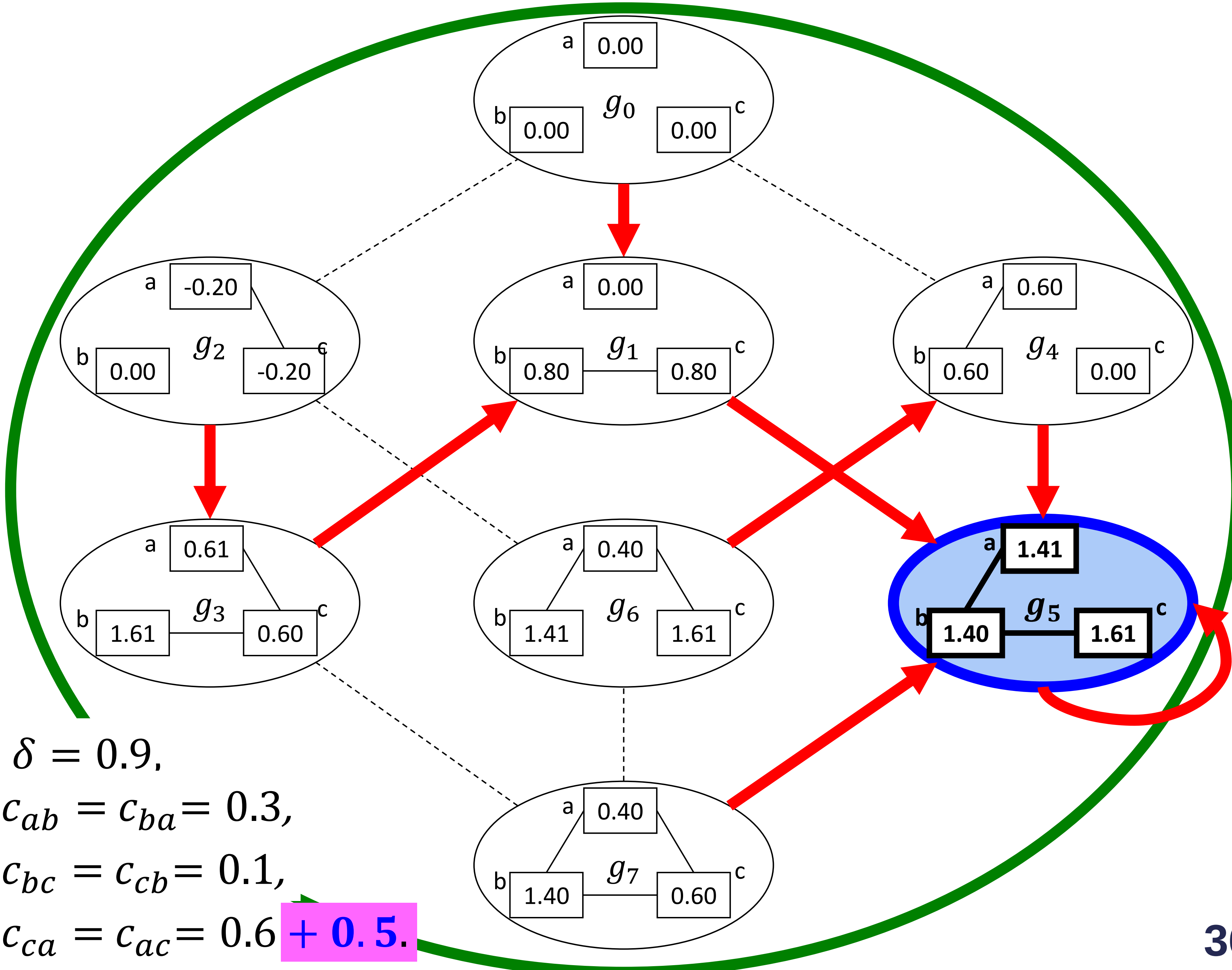


$\delta = 0.9,$

$c_{ab} = c_{ba} = 0.3,$

$c_{bc} = c_{cb} = 0.1,$

$c_{ca} = c_{ac} = 0.6 \, \mathbf{+ 0.5}.$



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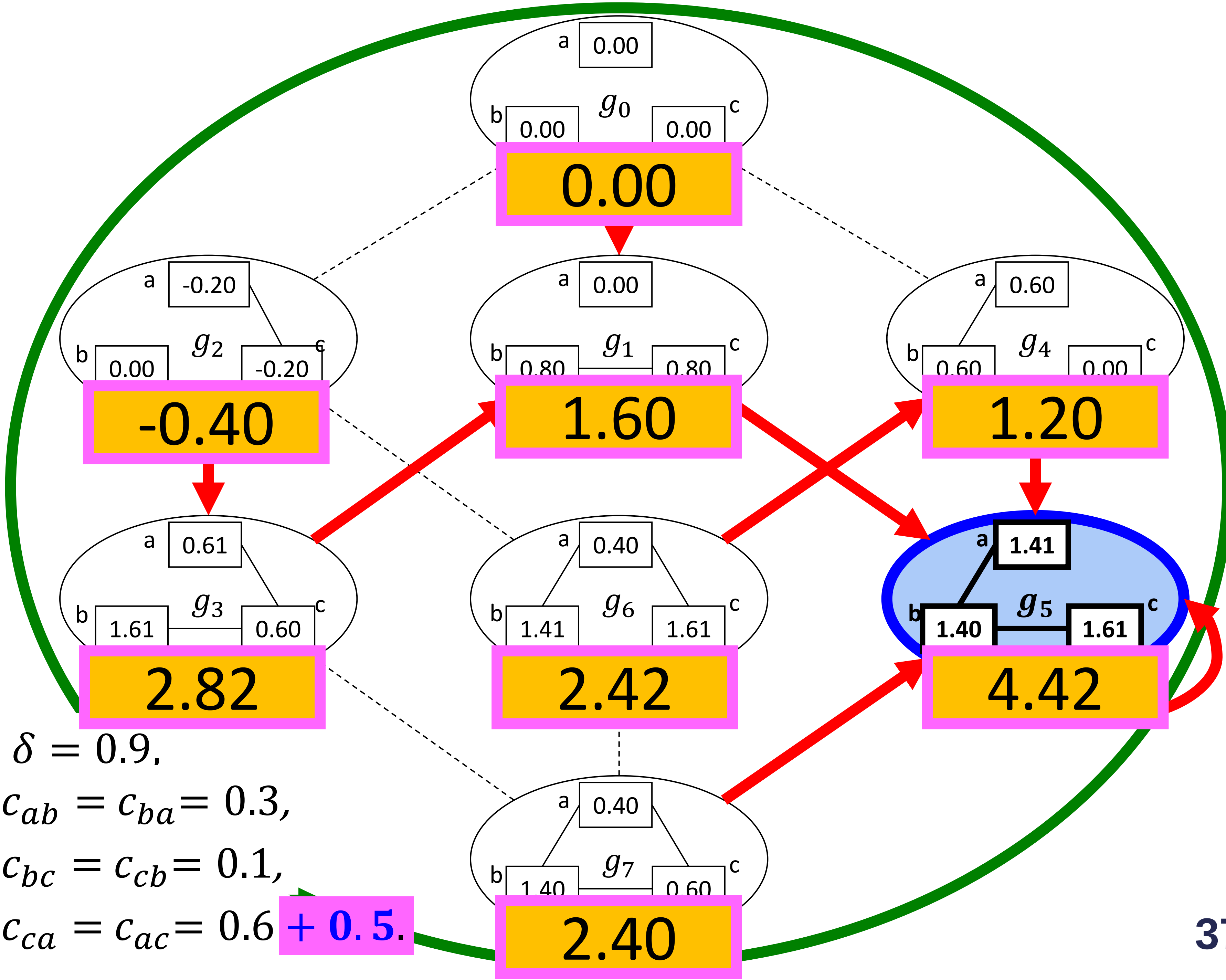

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