EVOLVING UNDER SMALL DISRUPTION

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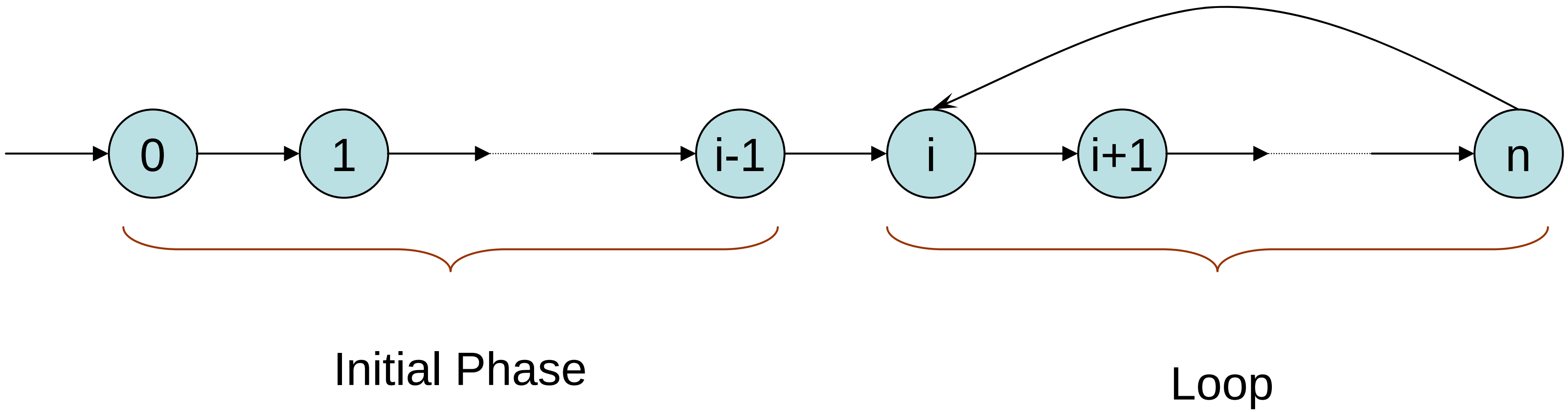
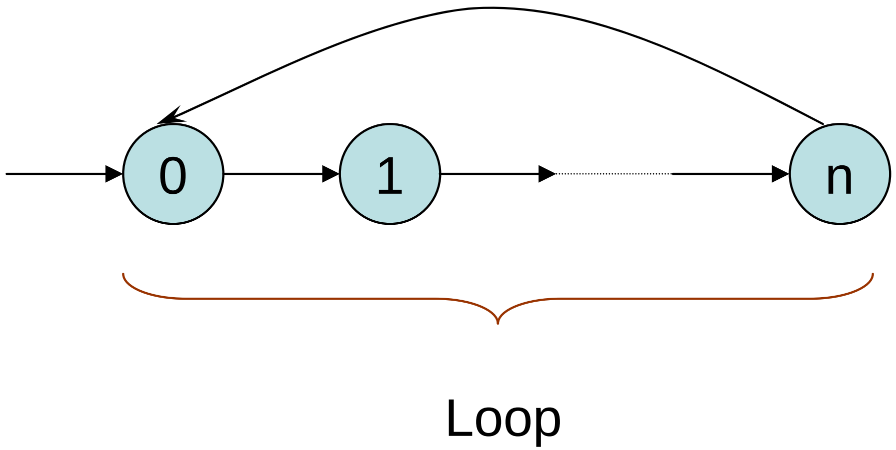
Unary Deterministic Finite Automata (UDFA)



Cyclic Unary Deterministic Finite Automata (CUDFA)



Representing CUDFAs and CURLs

- A CUDFA is represented as a word $w \in \{0,1\}^+$ (where ones are accepting states) and it will be considered as a genotype.

Let $w = x_1 x_2 ... x_n$ be a CUDFA, we set $B(w) = \{i \mid x_i = 1\}$.

- The regular language accepted by a CUDFA w is represented as an infinite set of disjoint successions of natural numbers

$$M = \{\{b_i + |w| k\}_{k \in \mathrm{N}}\}_{i=1,...,m}$$

where $b_i \in B(w)$ for any $i = 1,..., m$. This language will be considered as the corresponding phenotype of the CUDFA.

Some Bioinspired Operators over CUDFAs

Let $V = \{0,1\}$, $T(m, p) = \{w \mid w = (x_1 x_2 ... x_m)^p, x_i \in V \text{ for } 1 \le i \le m\}$ for $m, p \in \mathbb{N}^+$ and $h : V \to V$ be a mapping with $h(1) = 0$ and $h(0) = 1$. For any natural numbers $n, m, p > 0$, i with $1 \le i \le n$, $q > 1$ and $y \in V$, we define the operators:

- addition $A_{i,y} : V^n \to V^{n+1}$, $A_{i,y}(x_1 x_2 ... x_n) = x_1 x_2 ... x_i y x_{i+1} ... x_n$
- partial copy $PC_p : T(m, p) \to T(m, p+1)$, $PC_p((x_1 x_2 ... x_m)^p) = (x_1 x_2 ... x_m)^{p+1}$
- elimination $E_i : V^n \to V^{n-1}$, $E_i(x_1 x_2 ... x_n) = x_1 x_2 ... x_{i-1} x_{i+1} ... x_n$
- partial elimination $PE_q : T(m, q) \to T(m, q-1)$, $PE_q((x_1 x_2 ... x_m)^q) = (x_1 x_2 ... x_m)^{q-1}$
- mutation $M_i : V^n \to V^n$, $M_i(x_1 x_2 ... x_n) = x_1 x_2 ... h(x_i) ... x_n$

Disruption of an Operator over a CUDFA

- For two successions $A = \{a + bn\}_{n \in \mathrm{N}}$ and $B = \{c + dk\}_{k \in \mathrm{N}}$, the overlap $ISO_{A,B}$ of A and B (for Infinite Successions Overlap) is defined as:

$$ISO_{A,B} = \begin{cases} \dfrac{\gcd(b,d)}{d} & \text{if } A \cap B \neq \emptyset \\ 0 & \text{in other case} \end{cases}$$

- Let M and N be two CURLs, and let n be the number of successions of M. We define the overlap $URLO_{M,N}$ of M with N (for URLs Overlap) as:

$$URLO_{M,N} = \begin{cases} \dfrac{1}{n} \sum\limits_{\substack{A \in M \\ B \in N}} \mathrm{ISO}_{A,B} & \text{if } M \cap N \neq \emptyset \\ 0 & \text{in other case} \end{cases}$$

- Let $w \in V^+$ be a CUDFA and $O \in \mathcal{M} \cup \mathcal{A} \cup \mathcal{E} \cup \mathcal{PC} \cup \mathcal{PE}$ be an operator such that $O(w)$ is defined. Let L and L' be the CURLs represented by w and $O(w)$, respectively. We define the disruption $D(O,w)$ of the operator O over w as:

$$D(O,w) = (1 - URLO_{L,L'}, 1 - URLO_{L',L})$$

Disruption of the Operators

Lemma 2. Let $w \in V^+$ be a CUDFA. The CURLs represented by w and by w^n, with $n \in \mathrm{N}$ and $n > 1$ are the same.

Corollary 1. Let $w \in V^+$ be a CUDFA. The CURLs represented by w^n and w^m, $n, m \in \mathrm{N}$ and $n, m > 1$, coincide.

Corollary 2. For any $p, q > 1$, PC_p and PE_q are not disruptive operators.

Lemma 3. Let $w \in V^+$ be a CUDFA and i a natural number with $1 \le i \le |w|$. If $|w|_1 = m$, then

- $D(M_i, w) = (0, \frac{1}{m+1})$ if we mutate a zero into a one,
- $D(M_i, w) = (\frac{1}{m+1}, 0)$ if we mutate a one into a zero.

Disruption of the Operators

Lemma 4. For any CUDFA $w \in V^+$ with $|w|_1 = m$, any natural number i with $1 \leq i \leq |w|$, and any $y \in V$,

$$D(A_{i,y}, w) = (1 - \frac{m+y}{|w|+1}, 1 - \frac{m}{|w|})$$

Lemma 5. Let $w \in V^+$ be a CUDFA, $|w|_1 = m \geq 1$, i a natural number with $1 \leq i \leq |w|$, and *y* the i-th letter of w. Then

$$D(E_i, w) = (1 - \frac{m-y}{|w|-1}, 1 - \frac{m}{|w|})$$

Small Disruptions and Iterated Application of Operators

Let the CUDFA $w \in V_i^+$, $\mathcal{O} \subseteq \mathcal{M} \cup \mathcal{A} \cup \mathcal{E} \cup \mathcal{PC} \cup \mathcal{PE}$, and a real number λ, $0 < \lambda < 1$ be given.

- We say that a word v can be obtained with a disruption strictly less than λ from w using $\mathcal{O}$ if there exist operators $O_1, O_2, ..., O_p \in \mathcal{O}$, $p \geq 0$, such that
  - $v = O_p(O_{p-1}...(O_2(O_1(w)))...)$ and
  - $D(O_i, O_{i-1}(...(O_2(O_1(w))...)) < (\lambda, \lambda)$ for any $1 \leq i \leq p$.

- By $LD(w, \mathcal{O}, \lambda)$ we denote the set of all words v which can be obtained with a disruption strictly less than λ from w using $\mathcal{O}$.

Small Disruptions and Iterated Application of Operators

Theorem 1. Let $w \in V^+$ be a CUDFA and $0 < \lambda \leq \frac{1}{2}$ such that $\frac{1}{|w|_1 + 1} < \lambda$, and let $\mathcal{O} \subseteq \mathcal{M} \cup \mathcal{A} \cup \mathcal{E}$. Then

$$LD(w, \mathcal{O}, \lambda) = \{v \,\|\, v|_0 > 0, \frac{1}{|v|_1 + 2} < \lambda\} \cup \{1^m \mid m \geq 1\} \cup \{w\}.$$

<u>Proof</u>.

Let us suppose $|w|_0 = t$ and $|v|_0 = q$ for some $t, q \geq 0$. We choose

- $O_1, O_2, ..., O_t \in \mathcal{M}$, we mutate all the zeros of *w,* resulting $1^{|w|}$.
- Let $b = ||v| - |w||$.
  - If $|w| \leq |v|$, then $O_{t+1}, O_{t+2}, ..., O_{t+b} \in \mathcal{A}$, resulting $1^{|v|}$.
  - If $|w| > |v|$, then $O_{t+1}, O_{t+2}, ..., O_{t+b} \in \mathcal{E}$, resulting $1^{|v|}$.
- $O_{t+b+1}, O_{t+b+2}, ..., O_{t+b+q} \in \mathcal{M}$, we mutate all the positions in which $1^{|v|}$ has a one and *v* has a zero, resulting *v*.

Small Disruptions and Iterated Application of Operators

Corollary 3.

- Let $w \in V^{+}$ be a CUDFA and $0 < \lambda \leq \frac{1}{2}$ such that $\frac{1}{|w|_1 + 1} < \lambda$, and let $\mathcal{O} \subseteq \mathcal{M} \cup \mathcal{A}$. Then

$$LD(w,\mathcal{O},\lambda) = \{v \mid |w| < |v|, |v|_0 > 0, \frac{1}{|v|_1 + 2} < \lambda\} \cup \{1^m \mid m \geq 1\} \cup \{w\}.$$

- Let $w \in V^{+}$ be a CUDFA and $0 < \lambda \leq \frac{1}{2}$ such that $\frac{1}{|w|_1 + 1} < \lambda$, and let $\mathcal{O} = \mathcal{M} \cup \mathcal{PC}$. Then

$$LD(w,\mathcal{O},\lambda) = \{v \mid |w| < |v|, |v|_0 > 0, \frac{1}{|v|_1 + 2} < \lambda\} \cup \{1^m \mid m \geq 1\} \cup \{w\}.$$

Small Disruptions and Iterated Application of Operators

Theorem 2. Let $w \in V^+$ be a CUDFA and $0 < \lambda \leq \frac{1}{2}$, and let $\mathcal{O} = \mathcal{M} \cup \mathcal{PC} \cup \mathcal{PE}$. Then

$$LD(w, \mathcal{O}, \lambda) = V^+ \setminus \{0^m \mid m \geq 1\}.$$

Proof.
Let $|w| = m$, $|w|_1 = r > 0$, $|v| = n$ and $|v|_1 = s > 0$. Let $y = lcm(m, n)z,\ z \in \mathbb{N}^+$, we set $z' = \frac{y}{m}$ and $z'' = \frac{y}{n}$, with *z* sufficient large such that $\frac{1}{rz'} < \lambda$ and $\frac{1}{sz''} < \lambda$.

- $O_1, O_2, ..., O_{z'-1} \in \mathcal{PC}$, any O_i adds a copy of w, resulting $w^{z'}$.
- Let t be the number of positions in which $w^{z'}$ has a zero and $v^{z''}$ has a one. $O_{z'}, O_{z'+2}, ..., O_{z'+t-1} \in \mathcal{M}$, zeros are changed into ones, resulting $\overline{w}$.
- Let q be the number of positions in which $\overline{w}$ has a one and $w^{z''}$ has a zero. $O_{z'+t}, O_{z'+t+2}, ..., O_{z'+t+q-1} \in \mathcal{M}$, ones are mutated into zeros, resulting $w^{z''}$.
- $O_{z'+t+q}, O_{z'+t+q+2}, ..., O_{z'+t+q+z''-2} \in \mathcal{PE}$, any O_i cancels a copy of v, resulting v.

Small Disruptions and Iterated Application of Operators

Theorem 3. For any word *w* with $|w|_1 > 0$ and $0 < \lambda \leq \frac{1}{2}$,

$$LD(w, \mathcal{PC} \cup \mathcal{M} \cup \mathcal{A} \cup \mathcal{E}, \lambda) = \{v \,\|\, |v|_0 > 0, \frac{1}{|v|_1 + 2} < \lambda\} \cup \{1^m \mid m \geq 1\} \cup \{w\}.$$

<u>Proof</u>.

Let $|w|_1 = m \geq 1$, and let v be a word with $\frac{1}{|v|_1} < \lambda$. Then, there is a number $r \in N^+$such that $\frac{1}{mr} \leq \lambda$. Using *r*-1 times operators from $\mathcal{PC}$ which copy w, we get w^r.

Starting from w^r, we construct the same sequence of operators as in Theorem 1.

Conclusions

- The set of the **edit operators** has been **extended** by introducing the partial copy and partial elimination operators.

- We were able to **generate with low disruption** all words which correspond to **non-empty CURLs** by iterated applications of the operators **mutation, partial copy and partial elimination**.

- We have shown that **with the other set of operators** we can not generate with low disruption all words as before, but **the resultant set is also satisfactory** from a biological point of view.

Open Problems

- Searching algorithms to determine the minimal number of operators which transform with low disruption a given word into another given word.

- Studying whether the results presented in this work are also satisfied for more complex devices than CUDFA.

Thanks!