A robustness approach to study

metastable behaviours in a lattice-gas model of swarming

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Introduction

"Is **simple** also **robust**?"

Can one model a **robust** system with a **simple** model?

When is a model 'too' **simple** that it loses it **robust** quality?

Examples :

- space/time discretisation
- finite size
- use of periodic boundaries
- etc.

Introduction

"Is **simple** also **robust**?"

Robustness of models

How to know if a behaviour is ...

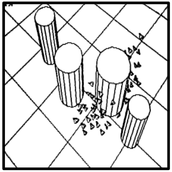
- **robust**– linked to an emergent phenomenon or
- **non-robust**– dependent on the model's hypotheses?

Loria Laboratoire lorrain de recherche en informatique et ses applications

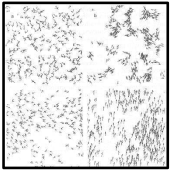
Introduction

"Is **simple** also **robust**?"

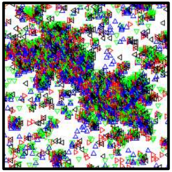
Example: the swarming behaviour



[Reynolds 86]



[Vicsek 95]

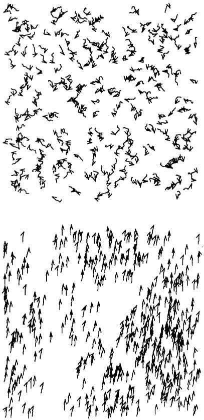


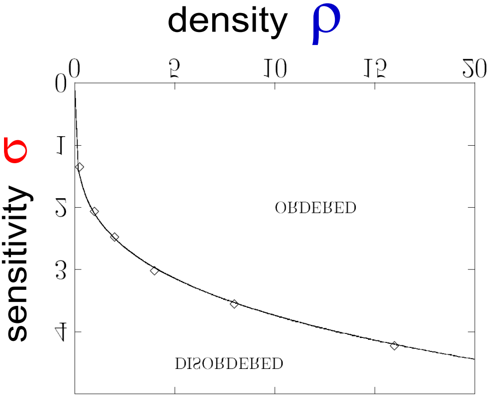
[Deutsch 95]

What is the behaviour?

In the continuous Vicsek model ...

Transition between organised/disorganised phases [Czirók *et al.* 97]



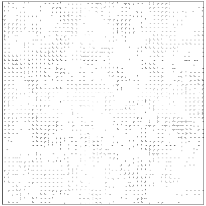


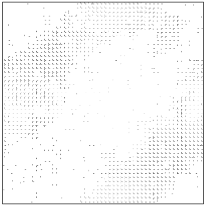
Can on reproduce this behaviour in a CA?

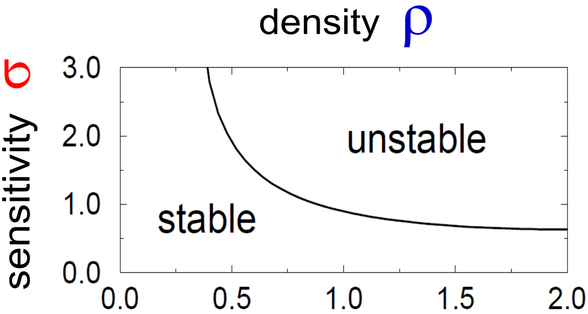
What is the behaviour?

In the lattice-gas Deutsch model ...

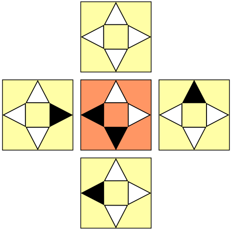
Proved using mean-field theory [Bussemaker *et al.* 97]







Lattice-Gas Cellular Automata (LGCA)



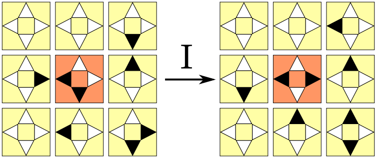
Cells are connected by *canals* though which *particles* may travel.

$$\begin{aligned}\mathbf{x}^t(c) &= \text{state of } c \text{ at time } t\\ &= [x_1; x_2; x_3; x_4] = [0; 1; 0; 1]\end{aligned}$$

LGCA	$\mathcal{A} = \{\mathcal{L}, \mathcal{N}, f_{\mathrm{I}}\}$
– a *grid*	$\mathcal{L} \subset \mathbb{Z}^2$
– a *neighbourhood*	$\mathcal{N} = \{\overset{n_1}{(1,0)}; \overset{n_2}{(-1,0)}; \overset{n_3}{(0,1)}; \overset{n_4}{(0,-1)}\}$
– an *interaction rule*	f_{I}

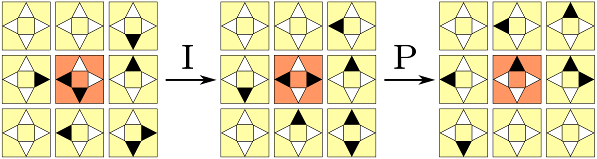
Dynamics of LGCA

1. Interaction : $\mathbf{x}^{\mathrm{I}}(c) = f_{\mathrm{I}}(\mathbf{x}^t(c), \mathbf{x}^t(c+n_1), \ldots, \mathbf{x}^t(c+n_4))$



(given by the model)

2. Propagation : $\mathbf{x}^{t+1}(c) = \left(x_1^{\mathrm{I}}(c-n_1), \ldots, x_4^{\mathrm{I}}(c-n_4)\right)$



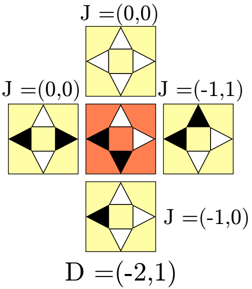
(determinist)

Interaction rule : favour the alignment

Probabilité de $\mathbf{x}_c^{\mathrm{I}}$

$$P(\mathbf{x}_c \to \mathbf{x}_c^{\mathrm{I}}) = \frac{1}{Z}\exp\left[\alpha.\mathbf{J}_c(\mathbf{x}^{\mathrm{I}}) \cdot \mathbf{D}_c(\mathbf{x})\right]$$

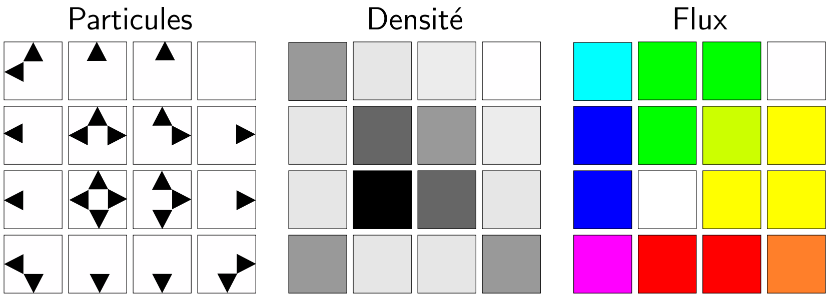
$\mathbf{J}_c(\mathbf{x}) \to$ cellular flux c, $\mathbf{D}_c(\mathbf{x}) \to$ director field c



						
σ	e^{σ}	$e^{3\sigma}$	$e^{-\sigma}$	$e^{-3\sigma}$	e^{0}	e^{0}
0.5	1.65	4.48	0.06	0.02	1	1
1.5	4.48	90	0.02	0.01	1	1
4	54.6	10^{5}	0.01	10^{-6}	1	1

(table of weights proportional to probabilities)

Visualisation



Order parameter

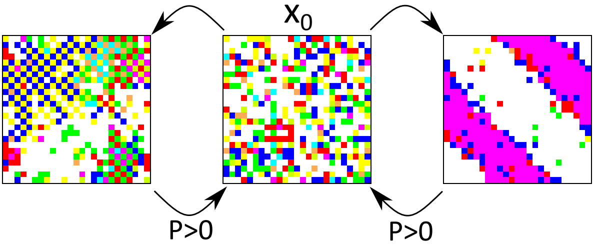
$$\gamma(\mathbf{x}) = \frac{1}{particles} \sum_{c \in \mathcal{L}} \frac{1}{4} J_c(\mathbf{x}) \cdot D_c(\mathbf{x})$$

Properties

- The Markov chain of the system is **recurrent**.

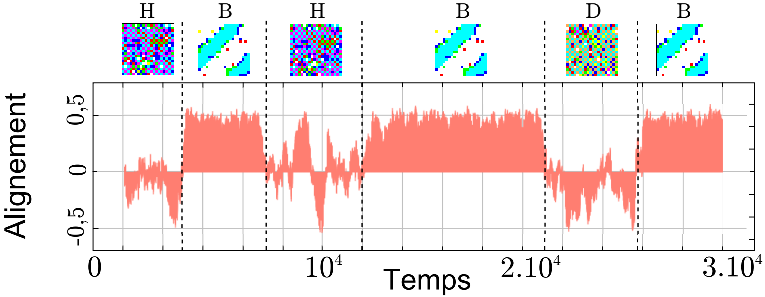
  In finite size:

  - Non-zero probability to return to initial condition.



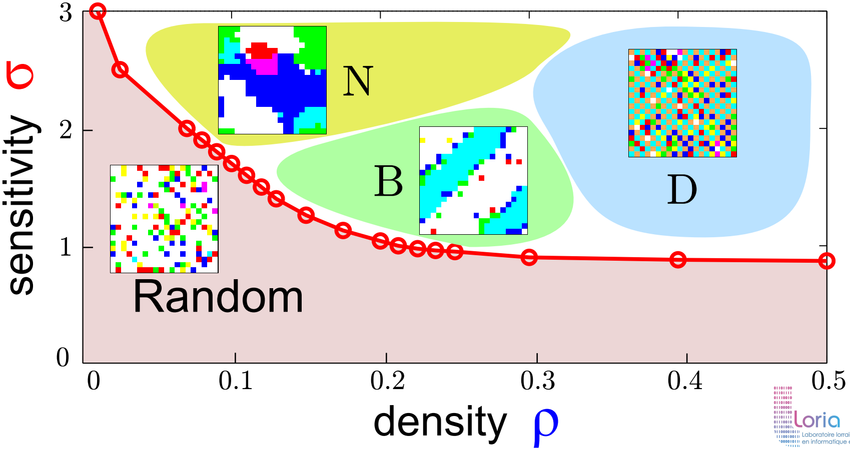
Properties

- The Markov chain of the system is **recurrent**.
- There exist several distinct **attractors**.
  - Any stable behaviour is temporary.

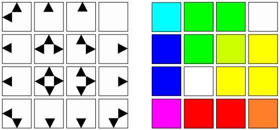


Properties

- The Markov chain of the system is **recurrent**.
- There exist several distinct **attractors**.
- These attractors are **metastable**.



Description of the patterns



Random

ρ small
σ small

Diagonal stripe

ρ medium
σ medium

Clusters

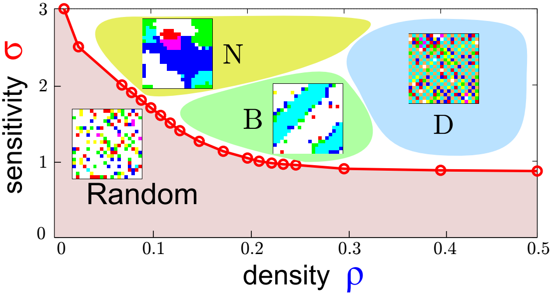
ρ medium
σ high

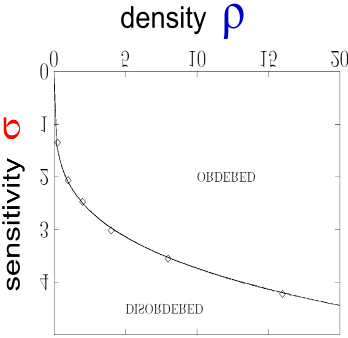
Checkerboard

ρ high
σ medium

First observations

and first surprise !





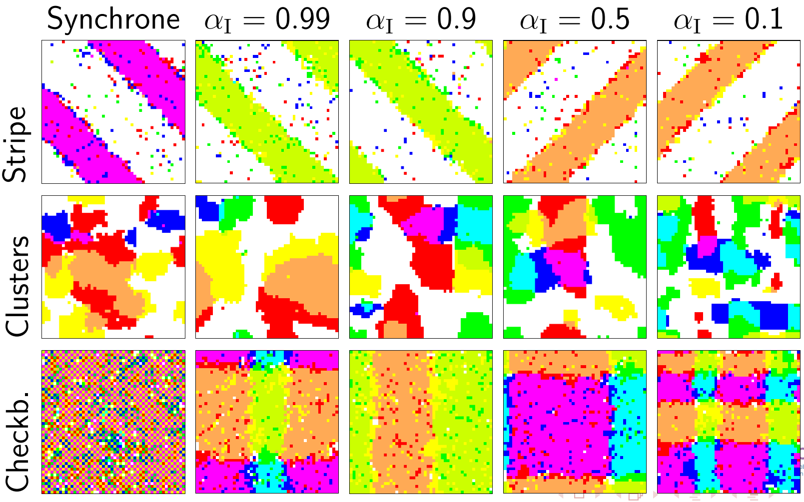
Lattice-gas model [Deutsch 95]

Continuous model [Czirók *et al.* 97]

1. organised/disorganised phase
2. detection of a new organised sub-structure !

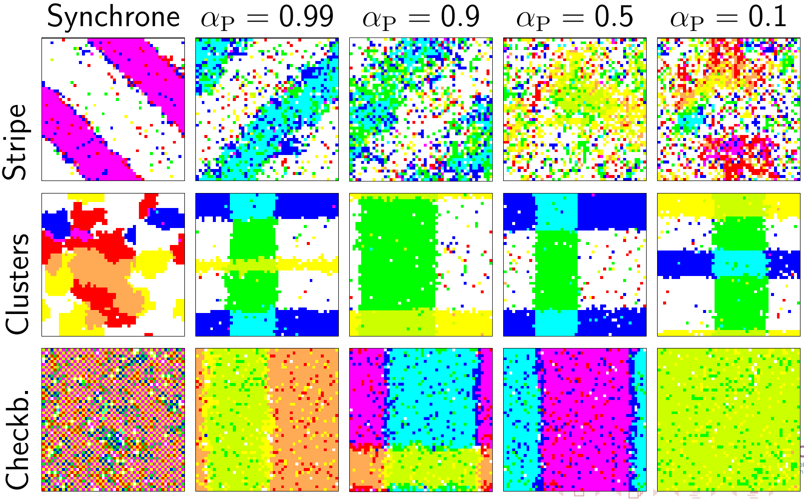
Asynchronous interaction [Bouré *et al.* 2012]

- The **interaction** step is applied with a probability α_I,
- ... otherwise cell is left unchanged.



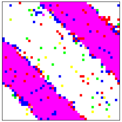
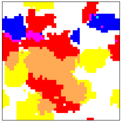
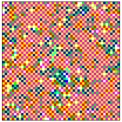
Asynchronous propagation [Bouré *et al.* 2012]

- The **propagation** step is applied with a probability α_{I},
- … otherwise *it's complicated* …



Results

Robustness to asynchronism

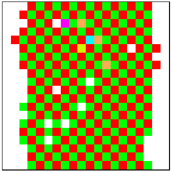
	Stripe 	Clusters 	Checkb. 
Async. interaction	**robust**	**robust**	**non**
Async. propagation	**a bit**	**non**	**non**

Robustness to lattice definition

In the definition, the grid is defined as $\mathcal{L} \subset \mathbb{Z}^2$.
$\rightarrow$ Does it have an influence on the observed behaviour?

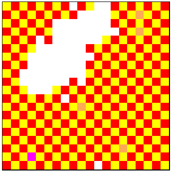
- A small grid ($L < 50$) may induce artifacts

‘belt’

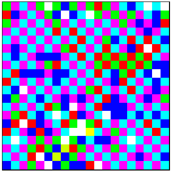


‘cross’

‘snake’



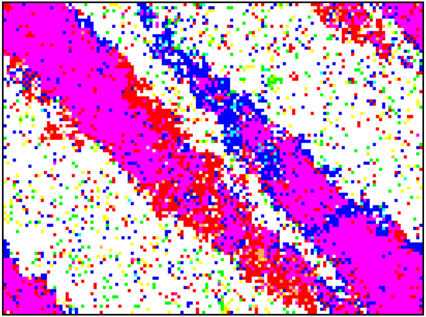
‘hybrid’



Robustness to lattice definition

In the definition, the grid is defined as $\mathcal{L} \subset \mathbb{Z}^2$.
$\rightarrow$ Does it have an influence on the observed behaviour?

- A small grid ($L < 50$) may induce artifacts
- A rectangular grid distrubs the regularity of the stripe.



ex. : grille de 135 par 100

Robustness to lattice definition

In the definition, the grid is defined as $\mathcal{L} \subset \mathbb{Z}^2$.
$\rightarrow$ Does it have an influence on the observed behaviour?

- A small grid ($L < 50$) may induce artifacts
- A rectangular grid distrubs the regularity of the stripe.
- **Resonance effects** for asymptotic behaviours.
  - Long time $\rightarrow$ permanent behaviour
  - 'Too' long time $\rightarrow$ resonance effects

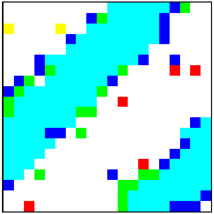
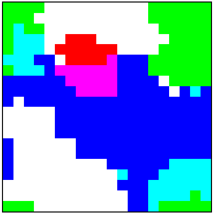
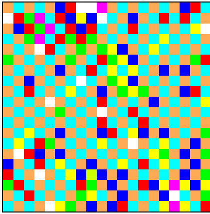
  Solution : increase space and time simultaneously

Finite $\neq$ infinite ?

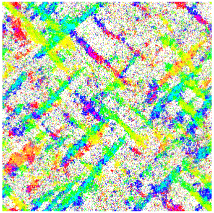
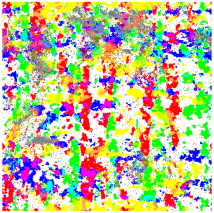
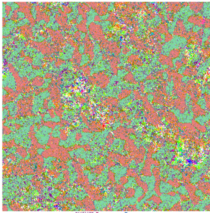
What is the behaviour for infinite lattices ?

Diagonal stripes — Clusters — Checkerboards

$T = 1000$
$L = \mathbf{100}$

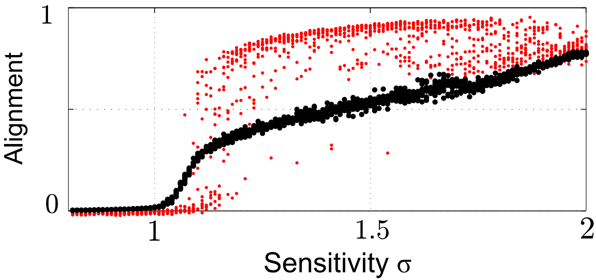




$T = 1000$
$L = \mathbf{1000}$

Finite $\neq$ infinite ?

What is the behaviour for infinite lattices ?

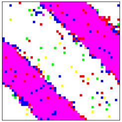
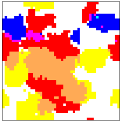
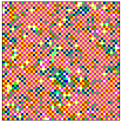
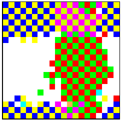


$$L = \begin{cases} 100 \text{ (red)} \\ 1000 \text{ (black)} \end{cases}$$

$$T = \quad 1000$$

Results

Robustness to grid definition

	Stripe	Clusters	Checkb.	Other
				
Async. interaction	**robust**	**robust**	**non**	–
Async. propagation	**a bit**	**non**	**non**	–
Grid size	**robust**	**robust**	**robust**	**non**
Rectangular shape	**a bit**	**robust**	**robust**	–
Passage to the limit	**robust ?**	**robust**	**robust**	–

Concluding remarks

Questions ...

- What is the ‘unbounded’ behaviour of the model?
  - How to observe it? Quantitatively or qualitatively ?
  - How **robust** is it?

- What is the nature of the transitions ...
  - between the organised/disorganised phases ?
  - between the different patterns ?

Thank you for your attention.

(Flock of common starlings above Rome)