Experimental study on convergence time of elementary cellular automata under asynchronous update

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Contribution of Work

- Study on convergence of elementary cellular automata (ECA) towards fixed-point attractors under fully asynchronous update.
  - Experimental study on rate of growth of convergence time with respect to the size of automaton under fully asynchronous update.
  - Six classes of convergence time are identified and that are:

$O(1)$, $O(\log(\log n))$, $O(\log n)$, $O(n^{1/2})$, $O(n^2)$, $O(2^n)$

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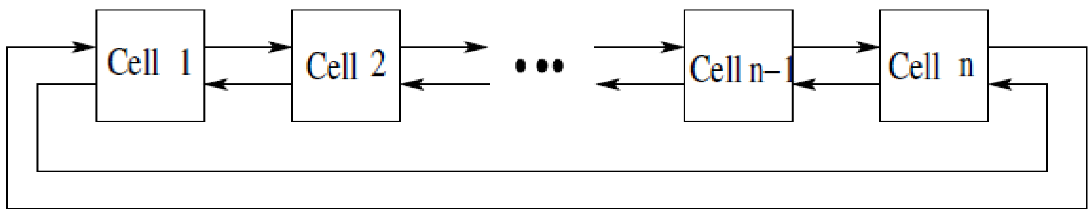
Introduction

CA Preliminaries

- A CA consists of a number of cells organized in the form of a lattice and can be viewed as FSM.
- Each cell stores a discrete variable at time t, called present state of cell.
- Next state can be obtained by present state of cell and its neighbors (left & right).
- In periodic boundary CA the left most & right-most cells are neighbors of each other.



periodic boundary CA

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Introduction (contd…)

- Next state of a cell at time t+1 can be obtained by:

$$S_i^{t+1} = f(S_{i-1}^t, S_i^t, S_{i+1}^t)$$

Where,

f : next state function
S^t_{i-1} : present state of left neighbor of cell at time t
S^t_i : present state of cell (self) at time t
S^t_{i+1} : present state of right neighbor of cell at time t

- The collection of states of cells at time 't' is the present state of CA
- Cell with neighbors (left and right) form 2^3=8 patterns.
  so, there are 2^8=256 possible rules in 3- neighbor CA.
- The decimal equivalent of 8 out puts is called rules of CA
- Each column of the first row of table is called RMT (rule min term)

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RMT

PS :	111	110	101	100	011	010	001	000	*Rule*
(*RMT*)	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)	
(i)NS :	0	0	1	0	0	1	0	0	36
(ii)NS :	0	1	0	0	1	1	0	1	77
(iii)NS :	1	1	0	1	1	0	1	1	219

Introduction(contd…)

- Asynchronous Cellular Automata (ACA)
  - Cells are independent
  - Any no. of cells updated randomly at single time step
  - No central clock (Independent clock)
  - Decentralized structure

- For our work we consider the fully asynchronous CA (i.e. Single cell updated randomly)

Introduction (contd…)

- Active & Passive RMT

  - A RMT of a rule is called active RMT, if an ACA cell flips its state (0 to 1 or 1 to 0) when acting on this particular RMT, Otherwise, that particular RMT is called Passive RMT.

Example:

RMT 4 (100) of rule 60 is active where as RMT 1(001) is passive

Rule 60							
111 (7)	110 (6)	101 (5)	100 (4)	011 (3)	010 (2)	001 (1)	000 (0)
0	0	1	1	1	1	0	0

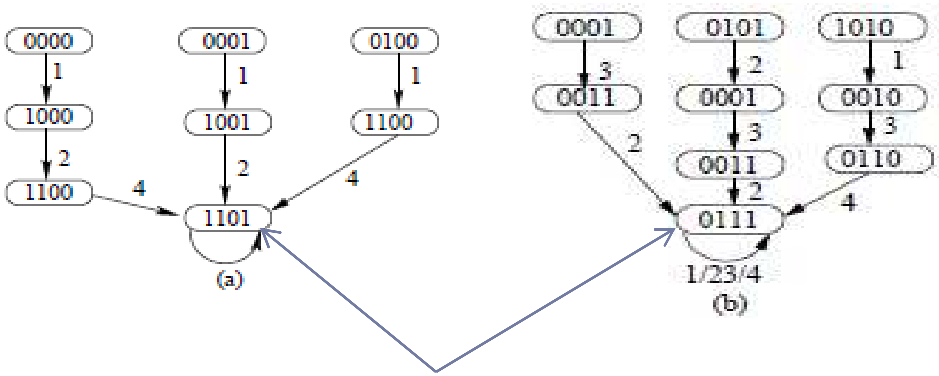
Introduction (contd…)

•Attractor:

If the state transition diagram of an ACA creates a loop, then the set of states which form that loop is called attractor

Fixed point attractor (single length):

- It is an ACA state, next state of which is the state itself for any update of cells
- In a fixed-point, all RMTs are passive



Fixed-point attractor

Partial state transition diagram for 4-cell of rule 219

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Characterization of ACA

- Convergence of ACA towards fixed-point attractor

Theorem:

Rule ‘R’ ACA converges to fixed-point attractor if one of the following condition is verified.

(i) RMT 0 (resp. RMT 7) of R is passive and RMT 2 (resp. RMT 5) is active.
(ii) RMTs 0, 1, 2 and 4 (resp. RMTs 3, 5 ,6 and 7) are passive and RMT 3 or 6 (resp. RMT 1 or 4) is active.
(iii) RMTs 1, 2, 4 and 5 (*resp.* RMTs 2, 3, 5 and 6) are passive.

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146 ACA (out of 256) are identified, that Converges to fixed-point attractor during their dynamic evolution.

Characterization of ACA (Contd…)

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From the theorem

Rules of ACA that always converge to fixed-point attractor

0	2	4	5	8	10	12	13
16	18	24	26	32	34	36	40
42	44	48	50	56	58	64	66
68	69	72	74	76	77	78	79
80	82	88	90	92	93	94	95
96	98	100	104	106	112	114	120
122	128	130	132	133	136	138	140
141	144	146	152	154	160	161	162
163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178
179	180	181	182	183	184	185	186
187	188	189	190	191	192	194	196
197	200	202	203	204	205	206	207
208	210	216	217	218	219	220	221
222	223	224	225	226	227	228	229
230	231	232	233	234	235	236	237
238	239	240	241	242	243	244	245
246	247	248	249	250	251	252	253
254	255						

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Estimating average convergence time

- For finding the average convergence time of an n-cell ACA, the convergence time for all possible states are considered.

- Average convergence time for an n-cell ACA is

$$\overline{X_{2^n}} = \frac{\sum_{i=1}^{2^n} X_i}{2^n}$$

Where,

X_i is the convergence time for i^{th} initial state.

2^n is the total no. of states for an n-cell ACA.

- But it is difficult to calculate the average convergence time for large size CA considering all possible states.

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- Hence, m states (out of 2^n states) are chosen randomly. So, estimated average convergence time is

$$\widehat{X_1} = \frac{\sum_{i=1}^{m} X_i}{m}$$

- To get better estimation, another 'm' samples are chosen randomly and calculate the average convergence time, considering total 2m states.
  Hence,

$$\widehat{X_2} = \frac{1}{2}\left(\widehat{X_1} + \frac{1}{m}\sum_{i=1}^{m} X_i\right)$$

Similarly, we can calculate, $\widehat{X_3}, \widehat{X_4}, \cdots, \widehat{X_k},$

Where,

$$\widehat{X_k} = \frac{k-1}{k}\left(\widehat{X_{k-1}} + \frac{1}{m}\sum_{i=1}^{m} X_i\right)$$

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- We declare $\widehat{\overline{X_k}}$ is the estimated average convergence time, if

$$|\widehat{\overline{X_k}} - \widehat{\overline{X_{k-1}}}| < \delta$$

Where, δ is a very small threshold value

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Method to find the rate of growth

- To find the rate of growth of convergence time we have followed the hybrid iterative refinement method of empirical curve bounding technique.

- Assuming the execution time (t) follows the power rule

$$t \approx kn^a$$

- The co-efficient 'a' can be found by taking empirical measurement of run time $\{t_1, t_2\}$ at some input size $\{n_1, n_2\}$. So,

$$a = \frac{log(t_2/t_1)}{log(n_2/n_1)}$$

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- We have experimented all 146 ACA several times varying the CA size.

- A series of coefficient ‘a’ can be found considering two consecutive average convergence time.

- From these several values of ‘a’ we find the rate of growth of convergence time by applying the hybrid iterative refinement method of empirical curve bounding technique.

- From the experimentation, we observed that, after a certain CA size the growth rate of convergence time always lies under some upper bound.

- we represent the rate of growth of convergence time of ACA in big-oh notation.

Experimental Results

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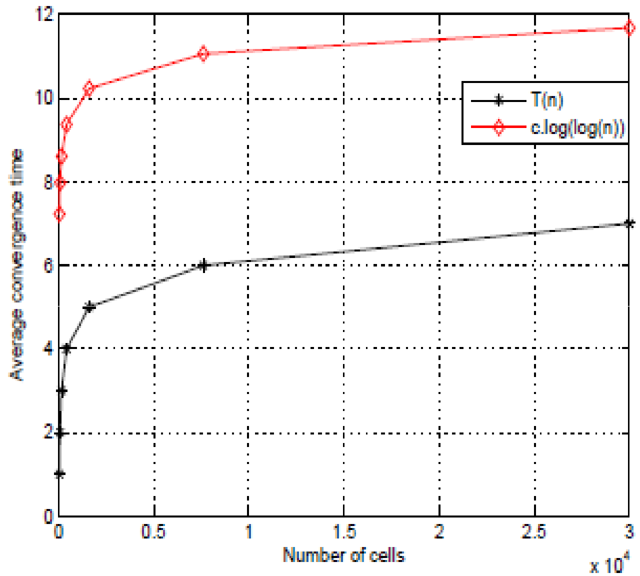
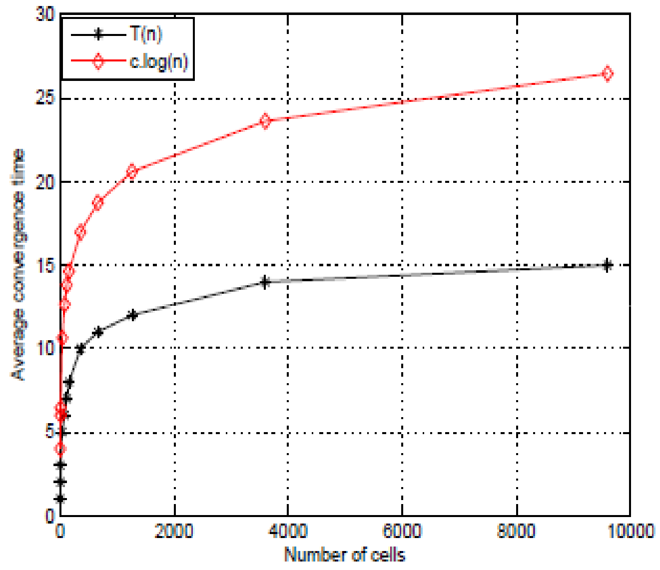
- Six classes of convergence time are identified.
  these are O(1), O(log(log n)), O(log n), $O(n^{1/2})$, $O(n^2)$, $O(2^n)$

Rate of growth	ACA
O(1)	204
O(log(log n))	0, 4, 5, 8, 12, 13, 32, 64, 68, 69, 72, 76, 77, 79, 93, 94, 128, 132, 133, 160, 164, 200, 205, 207, 218, 221, 222, 223, 232, 235, 236, 237, 239, 250, 251, 253, 254, 255
O(log n)	2, 10, 16, 18, 24, 34, 36, 42, 40, 44, 48, 50, 56, 66, 78, 80, 92, 95, 96, 98, 100, 104, 112, 130, 136, 140, 141, 144, 162, 168, 171, 172, 175, 176, 179, 183, 185, 186, 187, 189, 190, 191, 192, 196, 197, 202, 203, 206, 216, 217, 219, 220, 224, 227, 228, 234, 238, 231, 233, 241, 242, 243, 245, 246, 247, 248, 249, 252
$O(n^{1/2})$	26, 58, 74, 82, 88, 106, 114, 120, 163, 167, 169, 173, 177, 181, 225, 229,
$O(n^2)$	138, 146, 152, 170, 174, 178, 182, 184, 188, 194, 208, 226, 240, 244, 230
$O(2^n)$	90, 122, 154, 161, 165, 166, 180, 210

- Graphs are also plotted for rate of growth with respect to CA size for each class of convergence time



(a)



(b)

(a) Rate of growth of convergence time of O(log (log n))

(b) Rate of growth of convergence time of O(log n)

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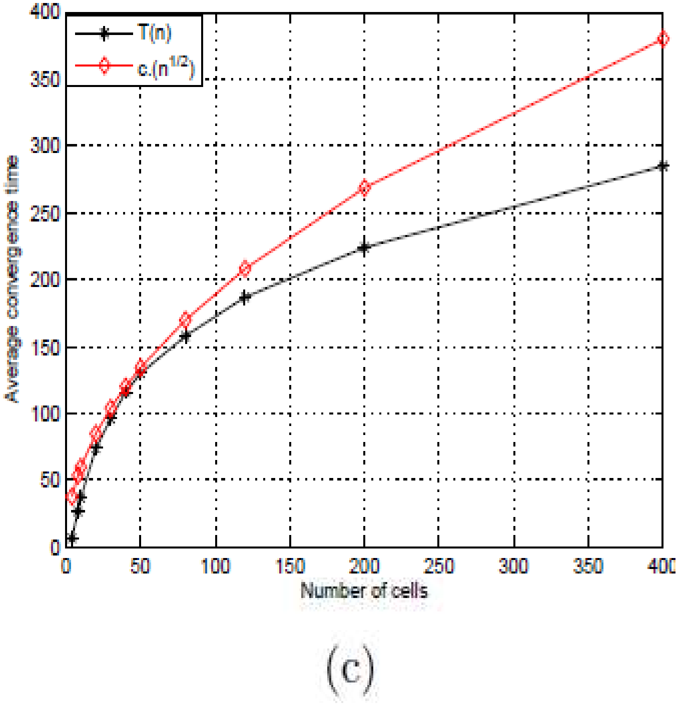

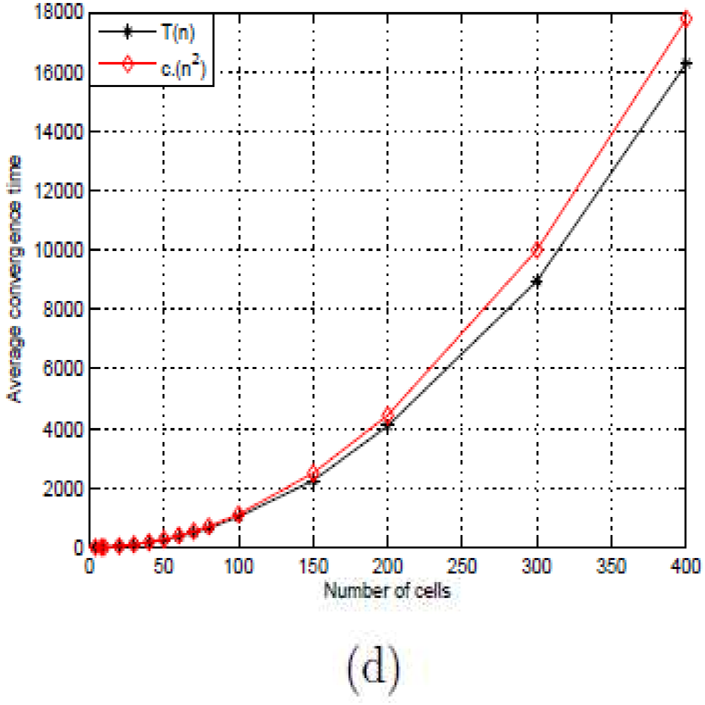

(c) Rate of growth of convergence time of $O(n^{1/2})$

(d) Rate of growth of convergence time of $O(n^2)$

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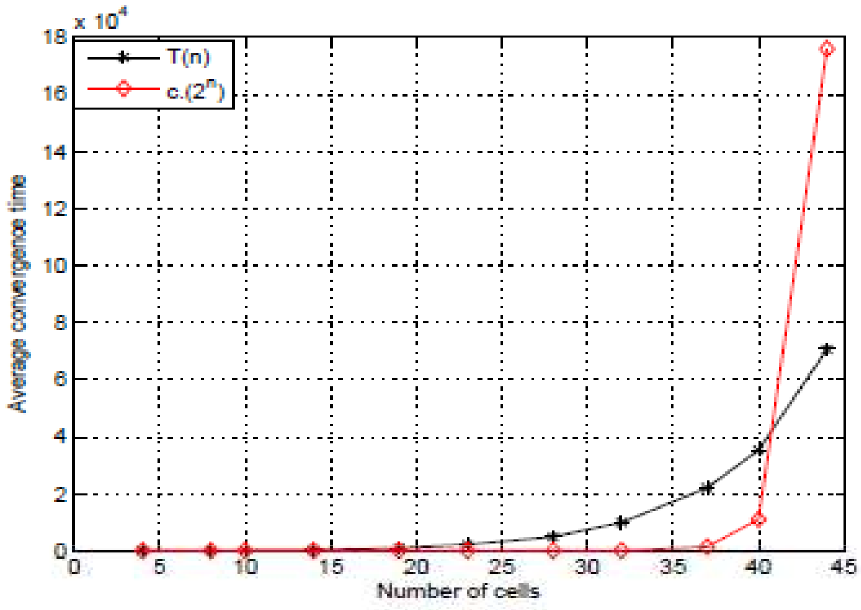

(e)

(e) Rate of growth of convergence time of $O(2^n)$

Conclusion & Future Work

- Characterization of ACA for the convergence toward fixed point attractor.
- Study of convergence time of ACA and six classes of convergence time are identified.

- Future Work:
- Study of convergence time of ACA considering different update patterns.
- Study of convergence time of ACA with multiple cell updates.

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Thank You

Questions ?

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